Multi-hour network planning based on domination between sets of traffic matrices

P. Pavon-Marino, B. Garcia-Manrubia and R. Aparicio-Pardo

Abstract—**In multi-hour network design, periodic traffic variations along time are considered in the dimensioning process. Then, the non coincidence of traffic peaks along the day or the week can be exploited. This paper investigates the application of the traffic domination relation between sets of traffic matrices to multi-hour network planning. Two problem variants are considered: a network with a static, and with a dynamic traffic routing. We derive a set of techniques for, given a multi-hour traffic demand potentially composed of hundreds of matrices, obtaining a traffic series with a smaller number of matrices. The traffic domination relation guarantees that the network designed for the simplified series is suitable for the original one. Also, we apply the domination relation to derive lower bounds to the network cost, and upper bounds to the suboptimality incurred by simplifying the traffic demand. The algorithms proposed are tested in a case of study with the Abilene network. In our tests, a long traffic series could be reduced to a small number of traffic matrices, and be effective for network planning.**

***Index Terms*—Network planning, multi-hour traffic, traffic domination.**

I. Introduction

MULTI-HOUR network design for various communication networks has been addressed by the research community over the years [1]-[14]. In the multi-hour (MH) network design model, the traffic demand is commonly represented as a series of traffic matrices, reflecting the traffic variation along a given period of time (typically days or weeks). The periodic nature of traffic has been confirmed with real traffic traces, such as the Abilene backbone network trace [15], making the expected traffic load in a network fairly predictable. Then, by introducing the traffic variation in the network design, it is possible to exploit the non-coincidence of the peak load moments in different parts of the network.

In this paper, we investigate two multi-hour problem variants: the MHSR (MH static routing) and MHDR (MH dynamic routing) problems. Given a series of traffic matrices, and a set of links in the network, the two problem variants are targeted to calculate the capacities in the links, and the routing pattern so that the demand is fully satisfied at any moment. In both cases, the link capacities are supposed to be fixed. The difference between the two problem variants lays on the routing pattern. While in the MHSR case the routing of the flows is constrained to be static, this constraint is not present in the MHDR case. Therefore, the MHDR case assumes an agile network, capable of changing the routing pattern according to e.g. a daily or weekly plan. In contrast, the MHSR is suitable for scenarios where the routing of the traffic cannot be changed to adapt the network to traffic variations.

We explore the application of the *traffic domination relation* presented in [16], to the scope of multi-hour traffic demands. Informally speaking, if a traffic demand H^1 dominates a demand H^2, then we can make the network design using H^1, with the guarantee that the capacities planned are suitable for the demand H^2. Two domination subtypes between traffic demands are applied: the *domination* relation and the *total domination* relation, for the MHDR and MHSR problems, respectively. We base our work on two propositions derived in [17] and [18] stating sufficient conditions for domination and total domination to occur, suitable for a wide variety of network planning problems. Based on this, our contribution consists of proposing a set of techniques for, given a traffic demand (potentially composed of hundreds or thousands of traffic matrices), computing simpler traffic series of *arbitrary* size that either (i) dominate or (ii) are dominated by the original one. A traffic series that dominates the original demand is applicable to feed a multi-hour network design algorithm associated to a particular network technology of interest. Then, as a novelty, we show how a traffic series *dominated* by the original demand can be used to derive lower bounds to the network cost for a wide range of cost models. Finally, a technique is proposed to calculate an upper bound to the cost sub-optimality that is being caused by planning the network using a simplified traffic series instead of the original one.

The rest of the paper is organized as follows. Section II connects the work in this paper with other related work. Section III introduces the notation and basic definitions, and propositions on the (total) domination relation. Then, a set of techniques applying the domination relation is presented in Section IV. Sections V and VI present algorithms applying

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these techniques to MHSR and MHDR problems, respectively. Results illustrating the suitability of the algorithms proposed are included in Section VII. Finally, Section VIII concludes the paper.

II. Related work

The planning of communication networks considering a time-varying demand has been intensely investigated for the different network technologies deployed in the last decades [1]-[14].

The first major case of success in multi-hour network design was the definition of the dynamic non-hierarchical routing (DNHR) in the mid 1980's for the telephone network [1]. It amplified the interest on investigating the planning models and algorithms to exploit the information on traffic variation, for the efficient network dimensioning. In [2], the problem of computing minimum cost link capacities in a circuit switching network satisfying a certain grade of service is studied. Only two predetermined candidate routes are assumed to be given for every communicating node pair. Recursive quadratic programming is applied to solve the resulting continuous nonlinear programming problem. In [4] the authors consider a multirate loss network with dynamic routing. They present a dimensioning model targeted to satisfy a given grade of service along different peak load periods. Then, a probabilistic admission control policy is proposed to decide on the rejection of a new call depending on the amount of idle capacity in the links. In [5] the authors deal with a multi-hour bandwidth packing problem that selects the calls to be routed on a capacitated telecommunication network, considering the waiting time of the demand before being satisfied. The capacity planning in a SONET/SDH network under a multi-hour demand is considered in [6].

The multi-hour network design in ATM networks is investigated in [7]. The author models the capacity dimensioning for multi-class dynamically reconfigurable networks. The algorithms presented are based on a Lagrangian relaxation of the programming model, and a subgradient optimization of the dual problem. The study in [7] is enriched in a more recent work [8], where the Lagrangian relaxation decomposition techniques are combined with a genetic algorithm approach.

In [11] the network design problem is addressed for IP networks based on shortest path routing protocols like OSPF. The paper considers network survivability requirements, and compares the merits of a set of proposed global and greedy heuristics. The multi-hour design for reconfigurable MPLS networks is considered in [12]. The algorithmic approach proposed by the authors deals with three subproblems separately: topology design, capacity dimensioning and traffic routing. In [14], the capacity upgrading of the links in a communications network required to satisfy a changing traffic demand is investigated. A measure for network performance evaluation called network lifetime is proposed, which intends to capture the intrinsic capabilities of the network topology to favor an efficient capacity expansion.

The multi-hour network design has been also studied in the scope of transparent optical networks. In [20] and [21] the Routing and Wavelength Assignment (RWA) problem is addressed under a multi-hour demand of optical connections. In [20] the authors propose a decomposition of the problem into the routing and wavelength assignment, while in [21] a scalable tabu-search based heuristic is presented.

This paper follows a different approach than the ones cited. We base our work on the concept of traffic domination relation presented by Oriolo in [16]. While the results in [16] are limited to the domination between individual traffic matrices, the author already motivates the interest on extending this relation for series of traffic matrices. As far as the authors know, three other works [17]-[19] after [16] investigated this approach.

In [18] the author presents some theoretical results extending the work in [16] for series of traffic matrices. The work in [18] addresses the application of the traffic domination to the MHSR and MHDR problems considering network survivability. In this line, some methods are proposed to take benefit of the traffic domination information to heuristically improve the performance of branch-and-bound algorithms for the targeted problem. In [19], the authors investigate the application of the traffic domination for the MHDR and MHSR planning in optical networks. The authors propose to convert the traffic series of optical connection demands into one individual traffic matrix, and plan the network according to it. Finally, in [17] the authors elaborate on necessary and sufficient conditions for total domination to occur.

The work presented in this paper takes benefit of the sufficient conditions for traffic (total) domination presented in [17] and [18]. A more general and more powerful method for network planning is proposed where the simplified traffic series can have an *arbitrary* number of matrices. In addition, a significant novelty of this paper is the utilization of the traffic (total) domination relation to derive lower bounds to the network cost, and derive upper bounds to the suboptimality caused by planning the network with the simplified traffic demand.

III. Preliminaries

A. Notation

Let N be the set of nodes in a network graph $G(N,E)$, being E the set of unidirectional links in the network. The initial and end nodes of link e will be denoted by $a(e)$ and $b(e)$, respectively. Also, we denote as $\delta^+(n)$ and $\delta^-(n)$ the set of links outgoing from and incoming to node n respectively. Let D denote the set of traffic demands in the considered network. Each demand $d\in D$ is characterized by its end nodes $a(d)$ and $b(d)$, and its traffic volume h_d. The vector of traffic volumes for all demands is equal to $h=(h_d, d\in D)$ $(h\in R_+^{|D|})$. Note that in this notation, we use a vector representation of the concept of traffic matrix, with one coordinate per demand. This is a more general representation of the concept of traffic matrix (where one demand type would exist for each input-output node pair). However, in this paper the expressions *traffic vector* and *traffic matrix* can be safely assumed by the reader to refer to the same traditional concept of traffic matrix.

A MH traffic demand is a finite set of traffic vectors $H=\{h^1, h^2, \ldots, h^T\}$. Such a set can represent traffic vectors that have to be supported by the network at different time intervals $t=1,\ldots,T$. We denote as *conv* (H) the convex hull of H: the set of traffic vectors composed as a convex combination of the vectors in H. For every time interval t, its traffic volume for demand type $d\in D$ is denoted h_d^t, so that $h^t=(h_d^t, d\in D)$. A capacity allocation $u=(u_e, e\in E)$ in the graph $G(N,E)$ is a vector in $R_+^{|E|}$ that assigns a non-negative capacity u_e to every link $e\in E$. Both the traffic vector and the capacity vector are supposed to be measured in the same traffic units, which may depend on the underlying technology being modeled (e.g. Gbps, Erlangs). Given a network graph $G(N,E)$, and a set of demands D, we define a static flow allocation pattern as a vector $x=(x_{de}, d\in D, e\in E)\in R_+^{|D||E|}$, where each x_{de} represents the fraction of traffic volume of demand d traversing link e. Similarly, a dynamic flow allocation pattern is a vector $x=(x_{det}, d\in D, e\in E, t\in T)\in R_+^{|D||E||T|}$. We denote as x^t the flow allocation vector at time interval t, $x^t=(x_{det}, d\in D, e\in E)\in R_+^{|D||E|}$.

B. Sufficient condition for total domination

The techniques proposed in this paper can be applied to any instance of the so-called *generalized* MHSR problem, described by (1):

$$\text{Find } (u,x), u\in\Re_+^{|E|}, x\in\Re_+^{|D||E|}$$

$$\text{Min } g(u,x), \text{ s.t.} \quad \text{(1a)}$$

$$\sum_{d\in D} x_{de}h_d^t \le u_e, e\in E, t=1,\ldots,T \quad \text{(1b)}$$

$$\sum_{e\in\delta^+(n)} x_{de} - \sum_{e\in\delta^-(n)} x_{de} = w_{dn}, d\in D, n\in N \quad \text{(1c)}$$

$$(u,x)\in Y \quad \text{(1d)}$$

Initially, no assumption is made about the cost function g in (1a). Constraint (1b) represents capacity requirements: capacity of every link must be sufficient to support the traffic load induced by any traffic vector in H. Constraint (1c) is the flow conservation constraint. The expression w_{dn} is given by (2):

$$w_{dn} = \begin{Bmatrix} 1 \text{ if } n=a(d), \\ -1 \text{ if } n=b(d), \\ 0 \text{ otherwise} \end{Bmatrix}, d\in D, n\in N \quad (2)$$

Finally, constraint (1d), where Y is some subset (proper or not) of $R_+^{|E|}\times R_+^{|D||E|}$, is a placeholder for possible extra constraints that could be defined for the problem, limiting the set of feasible solutions (u; x). As an example, non-bifurcated routing can be defined through $Y=R_+^{|E|}\times\{0,1\}^{|D||E|}$, and integral capacity through $Y=Z_+^{|E|}\times R_+^{|D||E|}$. When no additional constraints are assumed, i.e., when $Y=R_+^{|E|}\times R_+^{|D||E|}$, constraint (1d) is skipped.

Let H and H' be two MH traffic demands for a given generalized MHSR problem (1), defined by a set of demands D and a set Y of extra constraints.

Definition 1. We say that H totally dominates H' if, and only if every network design (u,x) suitable for MH demand H is also a valid design for the MH demand H' in problem (1).

The following proposition from [17],[18] states a sufficient condition for total domination to occur in generalized MHSR problems.

Proposition 1: In the same conditions described above, if for every vector $h'\in H'$ there exists a vector $h^*\in conv(H)$ such that $h^*\ge h'$, then H totally dominates H'.

Proof: See, [17].

We illustrate the geometrical interpretation of the concept of total domination between MH traffic demand sets with an example in Fig. 1. Let $H=\{h^1, h^2, h^3, h^4, h^5\}$ be a set of 5 traffic vectors for a network $G(N, E)$. To ease the graphical representation, let us suppose that all the traffic vectors in H are exactly equal, except for the different values observed in the coordinates of two particular demands $d_1,d_2\in D$ (which correspond e.g. to two different input-output node pairs). For a traffic vector h^i, the values in the d_1 and d_2 demands are represented in the pair of coordinates (a_i, b_i) in Fig. 1. Fig. 1 plots in a plane the (a_i, b_i) points, $i=1,\ldots,5$, corresponding to the 5 vectors of the set. Also, the points $P^i=(0, \max\{b_i\})$ and $P^{i'}=(\max\{a_i\}, 0)$ represent the maximum values in the two coordinates of the traffic vector h^i.

In this example, the vectors which are totally dominated by a traffic vector h^i are those whose (a_i, b_i) positions fall within the rectangle of vertices $\{(a_i, b_i), (0, b_i), (a_i, 0), (0, 0)\}$. For instance, traffic vector h^4 is totally dominated by h^1 and also is totally dominated by h^2. There is no single traffic vector in the set $\{h^1, h^2, h^3, h^4\}$ which totally dominates the vector h^5. However, note that h^5 is totally dominated by the set of traffic vectors $\{h^1, h^2, h^3\}$, as there is a convex combination of the vectors $\{h^1, h^2, h^3\}$ that totally dominates h^5. In the figure, we see that the traffic vector $0h^1+0.5h^2+0.5h^3$ totally dominates h^5. The shaded area in Fig. 1 delimits the traffic vectors that are totally dominated by the set $\{h^1, h^2, h^3\}$. Considering the proposition 1, if the shaded area contained each of the traffic vectors of a set (e.g. set $\{h^4,h^5\}$), the set is totally dominated by the set $\{h^1, h^2, h^3\}$.

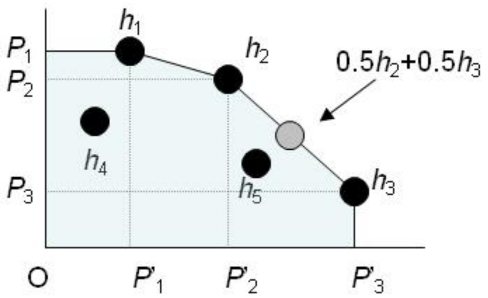


Fig. 1. Graphical representation of the total domination relation

C. Sufficient condition for domination

In (2) we define the *generalized* MHDR problem.

$$\text{Find } (u,x) = (u, x^1, ..., x^T), u \in \Re_+^{|E|}, x^t \in \Re_+^{|D||E|}, t = 1,...,T$$

$$\text{Min } g(u,x), \text{s.t.} \quad (3a)$$

$$\sum_{d\in D} h_d^t x_{\det} \le u_e, e \in E, t = 1,...,T \quad (3b)$$

$$\sum_{e\in\delta^+(n)} x_{\det} - \sum_{e\in\delta^-(n)} x_{\det} = w_{dn}, d \in D, n \in N, t = 1,...,T \quad (3c)$$

$$u \in Y \quad (3d)$$

The expressions in (3a-3c) are a trivial adaptation of (1a-1c) to the case in which the flow allocation x can change along time. A more significant difference appears in (3d). The extra constraints in (3) affect uniquely to the capacity variables. This is a technical requirement for the sufficient condition for traffic domination shown later to hold. Note that (3d) prevents the application of these techniques to the non-bifurcated routing variant of the MHDR problem.

Let H and H' be two MH traffic demands for a given generalized MHDR problem (3), defined by a set of demands D, and a set Y of extra constraints.

Definition 2. We say that H dominates H' if and only if every capacity vector u for which a valid time varying flow allocation x exists so that (u,x) supports the MH demand H, is also a valid capacity vector for the MH demand H' in problem, potentially using a different time varying flow allocation x'.

The following proposition from [18] states a sufficient condition for traffic domination to occur.

Proposition 2: For general MH traffic demands H and H', if $\forall h' \in H'$, there is a (potentially different) traffic matrix $h^* \in conv(H)$ so that h^* dominates h', then H dominates H'.

Proof: See, [18].

Proposition 3: Given two traffic vectors h and h', h dominates h' if and only if h' can be carried on top of h, seeing h as a capacity vector. Seeing h as a capacity vector means constructing a network $G(N,D)$ with one link e, for every demand $d \in D$, so that the capacity u_e of link e is given by the demand volume h_d.

Proof: See, [16].

IV. Application to the multi-hour planning

This section presents various techniques that take advantage of the domination relation between sets of traffic matrices, applied in the MHSR and MHDR problems (1) and (3). In Sections V and VI a set of algorithms based on these techniques will be detailed, and then tested in the results section of the paper.

A. Network design using an Upper Bound Traffic Demand

Multi-hour network design is fed by series of traffic matrices with a potentially large number of matrices. It can be of interest to replace such large set of traffic matrices by a smaller set with a lower number of elements. That smaller set could be used as an input to an algorithm solving the MHSR or MHDR problem, but allowing some potential suboptimality to arise.

The domination property helps us in this task. Given a potentially large set of traffic vectors H, we could search for a smaller set of vectors H^U (usually, $|H^U| << |H|$) that (totally) dominates H. Then, any design obtained in the (MHSR) MHDR problem for the reduced set H^U is a suitable planning design for the original traffic H.

If g is a cost function not directly dependent on the traffic vectors (for instance, an arbitrary function on the capacity planned u), then it holds that the cost of any design suitable for the H^U set is an upper bound to the optimal cost of the original problem. For this reason, we name the MH demand H^U as an *upper bound traffic demand* (UBTD) belonging to the *upper bound traffic demand* set of H ($H^U \in UBTD(H)$).

B. Cost lower bound using a Lower Bound Traffic Demand

Again, let H be a MH traffic demand and g be a cost function not directly dependent on the traffic matrices H. It is possible to use the domination property to derive lower bounds to the optimal cost of the MHSR or MHDR planning problems.

The method is based on calculating a new multi-hour traffic demand set H^L, so that H^L is dominated by the original set H (usually, $|H^L| << |H|$). Again, total domination is required in the MHSR problem, while domination applies to the MHDR problem. Thanks to the (total) domination property, every solution to the original problem is also a solution to the reduced problem, although the opposite statement is not necessarily true. Furthermore, under the same conditions showed in the previous subsection, the optimal cost of the MH problem for the demand H^L is a lower bound to the optimal cost of the original problem. Thus, the design obtained for the MH demand H^L is an infeasible solution for the original problem, but its optimal cost is a suitable lower bound for the optimal cost of the original problem. For this reason, we name the MH demand H^L as a *lower bound traffic demand* (LBTD), an element in the *lower bound traffic demand* set of H

($H^L \in LBTD(H)$).

C. Bound to the suboptimality caused by the transformation of the demand set

The subsection IV.A provides a technique for solving the MHSR and MHDR problems for a demand H, by first calculating a simplified demand $H^U \in UBTD(H)$, and then solving the target problem for the demand H^U. As a general rule, H^U demands with a small number of traffic vectors are preferred, as they imply less computational resources for solving the target problem. This raises the interest on measuring the maximum suboptimality we are incurring. That is, an upper bound to the extra planning cost that is directly caused by using the H^U demand instead of the original H demand. Herein we provide a technique for this case.

Let H^U and H^L be respectively an UBTD and a LBTD for a given MH traffic demand H, so that $|H^U| = |H^L|$. Let g be an arbitrary subadditive function of the planned capacity. That is: $g(u_1+u_2) \leq g(u_1)+g(u_2)$, u_1,u_2 capacity vectors in (1) or (3). Linear functions, norms and square roots are examples of such subadditive functions. Moreover, the conventional cost models that reflect the economy of scale principle for the capacity in the links fall into this category.

Let u_D^*, u_U^* and u_L^* be optimal capacities for the MH problems with H, H^U and H^L demands respectively, and let $c_D^* = g(u_D^*)$, $c_U^* = g(u_U^*)$ and $c_L^* = g(u_L^*)$ be their associated optimal costs. Then, the extra suboptimality we are incurring because of the simplification of the traffic demand is given by $\Delta c^* = c_U^* - c_D^*$.

In the following, we present a technique which is suitable for giving a bound to Δc^* even when only approximated solutions to the MHSR (MHDR) can be calculated for the simplified demands.

We say that a MH traffic demand H^E is an *excess traffic demand* (ETD) for the demands H^U and H^L, and we denote it as $H^E \in ETD(H^U,H^L)$, if and only if (i) $|H^E| = |H^U| = |H^L|$, and (ii) H^L+H^E (totally) dominates H^U. Note that the addition operation applies in this context to *sets of traffic vectors*. This assumes that the traffic vectors in the sets H^U, H^L and H^E are ordered in a series, and that the ith vector of the aggregated series H^L+H^E is equal to the sum of the ith vector of H^L (h^L_i) and the ith vector of H^E (h^E_i). In short words, H^E is a traffic demand, that when aggregated to the lower bound demand, (totally) dominates the upper bound demand. The following proposition shows that, once H^E is calculated, it can be used to bind the suboptimality Δc^*.

Proposition 4: The cost gap Δc^* is lower than the cost $c_E = g(u_E)$, being u_E a (non-necessarily optimal) capacity solving the MHSR (MHDR) problem for the excess traffic demand H^E.

Proof: The demand $H^{(L+E)} = H^L+H^E$ (totally) dominates H^U, and then also (totally) dominates the original demand H. The capacity matrix $u_{L+E} = u_L^*+u_E$ is able to solve the MH problem for the aggregated demand $H^{(L+E)}$. Then, the capacity matrix u_{L+E} is also a suitable design for the original problem. Consequently, the following set of inequalities hold:

$$c_U^* - c_D^* \leq g(u_L^*+u_E) - c_D^* \tag{4a}$$
$$g(u_L^*+u_E) - c_D^* \leq g(u_L^*+u_E) - c_L^*. \tag{4b}$$
$$g(u_L^*+u_E) - c_L^* \leq (c_L^*+c_E) - c_L^* = c_E. \tag{4c}$$

Inequality (4a) holds as the capacity matrix $u_{L+E} = u_L^*+u_E$ supports H^L+H^E and thus supports the UBD. (4b) holds as the cost of the LBD is a lower bound to the original cost, and (4c) as g is a subadditive function on the capacity.

D. General structure of the algorithms

Given a MH traffic demand H, we are interested in calculating convenient UBTD, LBTD and ETD demands composed of K matrices each. The algorithms proposed in this paper are based on (i) the distribution of the traffic matrices into K clusters, and then (ii) iteratively adapting the centroids of the clusters till they (totally) dominate/are dominated by the original traffic series. A similar sequence of four steps is proposed for the MHSR and the MHDR case:

Step 1: Partition the MH traffic demand H into K clusters $C^1, ..., C^K$ (with possibly different number of vectors each), according to the Criticalness Aware Clustering Algorithm (CritAC) [24]. We use the CritAC algorithm as it was specifically designed for clustering traffic vectors in communication networks. Nevertheless, our results have shown to be very similar using conventional clustering techniques like K-means. For each cluster C^k, $k = 1, ..., K$ we compute its representative traffic vector c^k, which we name *centroid*. The centroid is calculated as the convex combination of the vectors in the cluster with the same coefficient ($1/|C^k|$) for each term. The set of centroids is denoted as C (note that $|C| = K$).

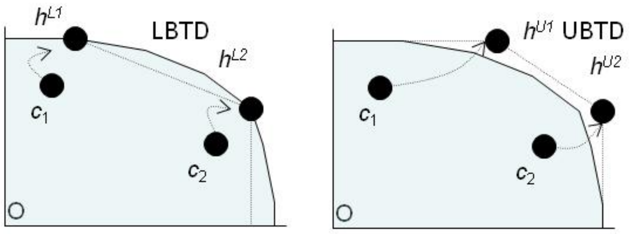
Step 2: Generate the MH demand H^U by iteratively modifying the K *centroids* until they totally dominate H.

Step 3: Generate the MH demand H^L by iteratively modifying the K *centroids* until they are totally dominated by H.

Step 4: Compute a convenient excess traffic demand $H^E \in ETD(H^U, H^L)$.

Fig. 2 illustrates graphically the steps 2 and 3 of the process in the MHSR case (total domination). The shaded area corresponds to the traffic vectors totally dominated by the original MH traffic demand. The target of the method is the modification of the initial centroids $\{c_1, c_2\}$, to obtain the LBTD $H^L = \{h^{L1}, h^{L2}\}$ (Fig. 2(a)) and the UBTD $H^U = \{h^{U1}, h^{U2}\}$ (Fig. 2(b)).

In the next sections, the algorithms in the steps 2, 3 and 4 are detailed in the MHSR and MHDR case.



(a) (b)
Fig. 2. Graphical representation of the centroid movement in the algorithm, (a) LBTD, (b) UBTD.

V. Algorithms for the MHSR Problem

In this section, we propose a set of effective algorithms for calculating upper bound, lower bound and excess traffic demands for a given MH demand H, in the planning problem MHSR.

A. Calculation of the upper bound demand

The algorithm we propose for calculating the set $H^U \in UBTD(H)$ iteratively moves the centroids $c^k \in C$. The final positions of the centroids constitute the searched traffic demand H^U. The pseudocode of the algorithm presented is shown in Fig. 3. In each iteration, the algorithm observes which vectors in the set H are not yet totally dominated by the set of K centroids C. We denote this not yet totally dominated set of vectors as H'. Then, the algorithm selects a coordinate $d \in D$ in a centroid c^k, and increases its value in a small quantity, with the intention of making one or more vectors in H' become totally dominated by the traffic vectors in C. As the coordinates of the centroids are always increased, and not decreased, a traffic vector that is totally dominated by the centroids in the iteration m will remain totally dominated by the updated centroids along the subsequent iterations.

In each iteration, we solve the LP formulation (5) for each of the traffic vectors $h^t \in H'$.

$$\text{Find } a^t = (a^{t1},...,a^{tK}) \in \Re_+^K$$

$$\text{Min} \sum\nolimits_{k=1,...,K} a^{tk}, \text{s.t.} \quad (5a)$$

$$h_d^t \le \sum\nolimits_{k=1,...,K} a^{tk} c_d^k, d \in D \quad (5b)$$

In (5) we search for the convex combination of centroids that dominates a traffic vector h^t, but without the constraint that forces the sum of the coefficients of the combination to be equal to one. Instead of that, we search for a solution that minimizes the sum of the coefficients of the combination $a^t=(a^{t1},\ldots,a^{tK})$. If this sum is equal or lower than one, the matrix h^t is already totally dominated, and is thus removed from H'. If not, the minimization process provides us with meaningful information. We are interested on selecting a matrix h^t in H' and a coordinate $d \in D$ in h^t, which may be more sensible to a modification of the centroids. In other words, the one for which a small increase in the centroids set can yield a larger approximation to make h^t totally dominated. The Lagrange multiplier λ_{td} associated to the constraint (5b) on the coordinate $d \in D$ of the traffic vector h^t, reflects this local sensitivity of the objective function. Thus, we may be interested on selecting the (t, d) duple with the highest multiplier.

Alternatively, if the maximum of the Lagrange multiplier is zero, we will choose the duple (t, d) that *minimizes* the slack variables sl_{td}. The slack variables sl_{td} indicate the margin between the coordinate d of h^t and the coordinate d of the convex combinations of centroids (5b). Then, a shorter margin represents a better chance to make the matrix h^t totally dominated.

Once we have obtained the duple (t, d), we choose the centroid $c^k \in C$ whose dth coordinate is to be increased: the one with the larger weight in the convex combination a^{tk}, among those candidate centroids for which $c^k_d < h^M_d$, $d \in D$. The vector h^M is the minimal vector which totally dominates the set H (that is, $h^M_d = \max\{h^t_d, t=1,\ldots,T\}$). In our tests the value of the step increase (Δh) is chosen to be the 1% of the average traffic in the coordinates of H.

B. Calculation of the lower bound demand

The method we propose for calculating the set $H^L \in LBTD(H)$ obtains each traffic vector $h^k \in H^L$ separately. In contrast to the iterative approach followed in the previous algorithm, the vector $h^k \in D^L$ is directly obtained from the centroid $c^k \in C$ by solving the LP (6).

$$\text{Find } a = (a^1,...,a^{|C^k|}) \in \Re_+^{|C^k|}, \Delta c^k = (\Delta c_d^k, d \in D) \in \Re_+^{|D|}$$

$$\text{Max} \sum\nolimits_{d \in D} \Delta c_d^k, \text{s.t.} \quad (6a)$$

$$c_d^k + \Delta c_d^k \le \sum\nolimits_{t=1,...,|C^k|} a^t h_d^t, d \in D \quad (6b)$$

$$\sum\nolimits_{t=1,...,|C^k|} a^t = 1 \quad (6c)$$

The algorithm searches for $\Delta c^k \ge 0$, maximum increase of the kth centroid, which still makes the new centroid $c^k+\Delta c^k$ being totally dominated by the set C^k, and thus by the original MH traffic demand H. Recall that the set C^k constitutes the subset of original arranged in the same cluster, and which has c_k as its representing centroid.

C. Calculation of the excess traffic demand

Given the UBTD $H^U=\{h^{U1},..,h^{UK}\}$ and the LBTD $H^L=\{h^{L1},..,h^{LK}\}$, we are interested on calculating an excess traffic demand $H^E=\{h^{E1},..,h^{EK}\}$ so that H^E+H^L totally dominates H^U. A suitable solution H^E MH traffic demand is given by (7):

$$h_d^{Ek} = \max\{0, h_d^{Uk} - h_d^{Lk}\}, d \in D, k = 1,..., K \quad (7)$$

Since each traffic vector $h^{Lk}+h^{Ek}$ totally dominates h^{Uk}, then H^E+H^L totally dominates H^U.

Algorithm to compute the UBTD (MHSR)

```
/* Iteration loop */
H' = H
while (H' not empty)
   for each h^t in H'
(*)   Formulation (5) (h^t,C)
   end
   H' = { h^t ∈ H' for which Σ_{k=1}^{K} a^{tk} > 1 }

   /* Select (t,d), d∈D,t=1,...,|H'|, so that domination
   of h^t is more sensitive to the modification of c^k_d */
   if max{λ'_td, d∈D,t=1,...,|H'|} > 0
      (t',d')=(t,d) for which λ_td is maximum
   else
      (t',d')=(t,d) for which sl_td is minimum
   end

   /* Select the centroid c^k whose d-th coordinate is
   going to be increased to dominate h^t */
   [k] = argmax(a^{t1},...,a^{tk}, so that c^k_d ≤ h^M_d')
   c^k_d' = min(c^k_d' + Δh, h^M_d')
end
```

Fig. 3. Pseudo-code of algorithm to compute the UBTD (MHSR problem).

VI. MHDR PROBLEM

A. Calculation of the upper bound traffic demand

The algorithm proposed in the MHDR case follows the same pseudocode shown in Fig. 3 for the MHSR problem. The only difference is tagged as (*) in Fig. 3: the formulation to be used to check the domination between the original MH traffic demand and the current set of centroids is now given by (8):

$$\text{Find } a^t = (a^{t1},...,a^{tK}) \in \Re_+^K,$$
$$x^t = (x_{d'dt}, d', d \in D) \in \Re_+^{|D||E|}$$
$$\text{Min} \sum_{k=1,...,K} a^{tk}, \text{s.t.} \quad (8a)$$
$$\sum_{d' \in D} h_{d'}^t x_{d'dt} \le \sum_{k=1,...,K} a^{tk} c_d^k, d \in D \quad (8b)$$
$$\sum_{d \in D} x_{d'dt} = 1, d' \in D \quad (8c)$$

Formulation (8) tries to find a positive combination h^* of the centroids in C, so that h^* seen as a capacity vector is able to carry the traffic in h^t. If a combination whose coefficients sum 1 or less is found, the h^t vector is already dominated, and thus removed from H'. Constraints (8b) and (8c) differ from (5b) by reflecting the domination relation requirements instead of the total domination relation ones.

B. Calculation of the lower bound traffic demand

We propose a computationally simple method composed of two consecutive steps, each one controlled by a linear program. First, each centroid $c^k \in C$ is scaled separately by using the LP formulation (9). This formulation searches for the lowest multiplicative factor $\beta^k \geq 0$ so that the traffic vector $c^{k\,\prime} = c^k/\beta^k$ becomes dominated by the set C^k. This problem can be formulated linearly by some simple transformations:

$$\text{Find } \beta^k \in \Re_+, a^k = (a^{kt}, h^t \in C^k) \in \Re_+^{|C^k|},$$
$$x^k = (x_{d'dk}, d'd \in D) \in \Re_+^{|D||D|}$$
$$\text{Min } \beta^k, \text{s.t.} \quad (9a)$$
$$\sum_{d' \in D} c_{d'}^k x_{d'dk} \le \sum_{h^t \in C^k} a^{kt} h_d^t, d \in D \quad (9b)$$
$$\sum_{t, h^t \in C^k} a^{kt} = \beta^k \quad (9c)$$

Note that in (9), the variable β^k does not appear dividing c^k in (9b). In its turn, it appears in (9c) as the total weight of the coefficients in the linear combination of traffic vectors in C^k. However, if both (9b) and (9c) are divided by β^k, we obtain an equivalent problem in which the (i) linear combination is convex, (ii) the resulting combination dominates c^k/β^k.

Let us denote as $c^{k\,\prime} = c^k/\beta^k$ the new position of the centroids. Now, a postprocessing step is performed independently for each centroid. The postprocessing uses the solution to the previous problem (9) together with the slack variables vector $sl = \{sl_d, d \in D\}$ to this problem:

$$sl_d = \sum_{h^t \in C^k} a^{kt} h_d^t / \beta^k - \sum_{d' \in D} c_{d'}^k x_{d'dk} / \beta^k, d \in D \quad (10)$$

The value sl_d represents the residual capacity that remains in the virtual links of the artificial capacity vector given by the convex combination of traffic vectors calculated, when it routes the demand vector given by the new centroid $c^{k\,\prime}$. We are interested in using this residual capacity to route extra traffic Δc^k that we want to add to the centroid $c^{k\,\prime}$. By doing so, we produce a LBTD with more traffic, and thus a better LBTD. LP formulation (11) implements the method described. Finally, the MH LBTD is computed as $H^L = \{h^E_k = c^k/\beta^k + \Delta c^k, k = 1, ..., K\}$.

$$\text{Find } \Delta c^k = (\Delta c_d^k, d \in D)$$
$$\text{Min} \sum_{d \in D} \Delta c_d^k, \text{s.t.} \quad (11a)$$
$$\sum_{d' \in D} \Delta c_{d'}^k x_{d'dk} \le sl_d, d \in D \quad (11b)$$

Note that in (11) the routing of the flows is the one already calculated in (9). Therefore, the x^k vector is an input parameter to (11) and not a decision variable.

C. Calculation of the excess traffic demand

Given the UBTD $H^U=\{h^{U1},..,h^{UK}\}$ and the LBTD $H^L=\{h^{L1},..,h^{LK}\}$, we are interested on calculating an excess traffic demand $H^E=\{h^{E1},..,h^{EK}\}$ so that H^E+H^L dominates H^U. The method we propose calculates separately each vector $h^{Ek}\in H^E$, $k = 1, ..., K$, solving the formulation (12).

$$\text{Find } h^{\mathrm{Ek}} = (h_d^{Ek}, d \in D) \in \Re_+^{|D|},$$

$$x^k = (x_{d'dk}, d', d \in D) \in \Re_+^{|D||D|}$$

$$\text{Min} \sum_{d\in D} h_d^{Ek}, \text{s.t.} \quad (12a)$$

$$\sum_{d'\in D} h_{d'}^{Uk} x_{d'dt} \le h_d^{Lk} + h_d^{Ek}, d \in D \quad (12b)$$

$$\sum_{d\in D} x_{d'dk} = 1, d' \in D \quad (12c)$$

Constraints (12b) and (12c) searches for a h^{Ek} that when added to h^{Lk}, dominates h^{Lk}. By repeating this for every k=1,…,K we obtain a valid excess traffic demand H^E.

VII. Results

This section presents some results targeted to illustrate the benefits of the techniques proposed. The algorithms were implemented in the MatPlanWDM tool [25] which links to the TOMLAB/CPLEX library [26] used to solve the MILP (Mixed-Integer LP) and LP formulations. As a case of study, we choose the planning of a hypothetical IP/MLPS network, built on top of the 11-node Abilene topology, with modular capacities. The capacities installed in the links must be a multiple of C Gbps. Since the resulting MHSR and MHDR planning problems fall in the definition of *generalized* MHSR and MHDR problems, the techniques proposed in this paper can be safely applied.

The traffic vectors used for the Abilene network were taken from the measures carried out in the TOolbox for Traffic Engineering Methods (TOTEM) project [15]. The sequence of vectors available in [15] spans several weeks. From this data, we averaged the values taken at the same time and day in the week to obtain a sequence representing the average week in 1-h time intervals (24 matrices per day, $T = 168$ matrices in total).

All the traffic values in the sequence were multiplied by a normalization factor (*nf*), dependent on a given parameter ρ.

$$nf(\rho) = \frac{n\cdot(n-1)\cdot\rho\cdot C}{\max\left\{\sum_{d\in D} h_d^t, t = 1,...,T\right\}} \quad (13)$$

where h^t_d is the traffic in Gbps from node $a(d)$ to node $b(d)$ during time interval t. Value *nf* was calculated such that the average traffic between two nodes in the most loaded time slot equals $\rho\cdot C$. Then, values of ρ<1 mean that the average traffic between two nodes in the most loaded time slot is below one module of capacity (low load scenario). In the contrary, large values of ρ (ρ>>1) means that the traffic load is high if compared to the modular capacity. Higher values of ρ make the planning problem more and more equivalent to its non-modular counterpart. The values tested in our study were ρ = {0.1, 1, 10}.

For both problem variants MHSR and MHDR, the cost function is considered to be linear with the total capacity installed. As a pre-processing step, we applied the method proposed in [19] for filtering out from the traffic demand those traffic matrices which are dominated by other traffic matrix in the MH demand. These traffic matrices can be safely eliminated from the planning problem since they are redundant. That is, removing them neither changes the set of feasible solutions to the planning problem, nor adds any suboptimality. The matrix filtering process reduced the number of traffic vectors to 127 (25% reduction) and 32 (80% reduction) for the MHNR and MHDR problems, respectively.

The non-redundant traffic series are used to generate the UBTD, LBTD and ETD demands for different reduced demand sizes K. The optimal capacities satisfying the UBTD and LBTD demands are obtained by an optimal MILP formulation. The solver was configured to find a solution within a 5% of optimality gap. The reason for searching for an optimal solution for the UB and LB demands is to permit a more precise evaluation of the suboptimality caused by the proposed reduction techniques. By doing so, the subopimality evaluation is not obscured by the suboptimality added to the process by a heuristic planner.

In Fig. 4(a)-(c) the total capacity requirements (in number of modular links) for the MHSR and MHDR problems are plotted for the load values ρ = {0.1, 1, 10} respectively. Each graph shows the evolution of the network cost in the MHSR and MHDR problems, solved for the UBTD and LBTD demands, for increasing sizes $K = \{1,2,3,4,5,6,7,8,9,10,15,20,25,30\}$.

The UBTD and LBTD optimal costs tend to approach in every case for increasing values of K, corresponding to more accurate approximations to the original traffic demand. However, in some occasions appearing in small values of K, increasing K in one unit resulted in slightly worse upper bounds or lower bounds. This is a natural consequence from the heuristic clustering-based approach for the calculation of the UBTD and LBTD demands.

Observing the evolution of the cost of the network designed from the UBTD demand, we see that the planning cost decreases more intensely in a first range of values of K, while the benefit from increasing the demand size K becomes marginal after it. This threshold could be situated in around K = 15 traffic vectors, roughly equal in the static and dynamic routing cases. If we focus on the quality of the lower bounds calculated from the LBTD demand, a similar analysis can be made. In this case, the lower bound cost significantly increases for higher values of K in the range [1, 10] approximately. After it, the quality of the lower bound improves marginally with K.

Fig. 5(a)-(c) plots the upper bounds to the suboptimality gaps. That is, the bounds to the extra cost incurred in the network planning, caused by using the reduced traffic demand instead of the original traffic demand. We compare the bounds calculated in the two different manners presented in this paper: (i) bound Δc_1, given by the difference between the UBTD and LBTD planning costs, (ii) bound Δc_2, given by the planning cost of the excess traffic demand (ETD). As justified in Section IV.C, this latter technique is targeted to those situations in which an optimal solution to the problem is not possible even in the reduced case. Nevertheless, in order to evaluate the quality of this bound we provide the planning cost of the ETD calculated by an exact MILP formulation. Then, the accuracy of the bound is not obscured by the added suboptimality caused by a heuristic-based planner. Note that the Δc_1 and Δc_2 provide an absolute cost gap. That is, a maximum added cost in monetary units. However, for the sake of clarity, the gaps in Fig. 5 are plotted as a percentage with respect to the cost of the UBTD.

The trend in the evolution of the gap Δc_1 is governed by the evolution of the costs for the UBTD and LBTD demands: a regular decrease in the gap is observed with increasing values of K, being more intense around the range $K\in[1, 15]$. Results are roughly independent from the network load. They are encouraging, as narrow suboptimality gaps can be guaranteed with moderate sizes of the reduced demand. As an example, $K = 15$ and $K = 20$ traffic vectors are enough to guarantee a suboptimality gap Δc_1 below 10% in the dynamic and static routing case, respectively. A more detailed analysis shows that in general, the Δc_1 gap is about 5% smaller in the MHDR problem, compared to the static routing case. In fact, the upper bound and lower bound coincide in the MHDR $\rho = 0.1$ for $K = 30$ traffic vectors case guaranteeing that the optimal solution to the original traffic demand can be obtained from the UBTD demand.

As stated in (4), the gaps Δc_1 directly calculated from the UBTD and LBTD costs are always tighter than those obtained from the excess demand (Δc_2). Fig. 5 shows that the Δc_2 curve is approximately a translation of the Δc_1 curve for higher values. In the low load condition, the best suboptimality gap Δc_2 was in the order of 30%, for both the static and dynamic routing cases. The results are better on medium and higher load situations. Cost gaps in the order of 10-20% could be calculated in those cases. K values from 20 to 30 are needed to obtain that accuracy.

If we compare the static routing and dynamic routing cases we see that, logically, the UBTD design cost is always better for the dynamic routing case. This reflects the savings in link capacities that can be obtained from the possibility of dynamically rerouting the traffic. Estimating such a maximum capacity saving is a relevant topic. Network operators should balance the capacity savings against the extra network operation cost, traffic disruption effects and extra signalling burden added by traffic rerouting. The techniques proposed in this paper can help in this evaluation.

Given a MH traffic demand H and a cost function g based solely on the network capacities, we are interested in the difference between the optimal costs in the static routing and dynamic routing cases: c^*_{MHSR}-c^*_{MHDR}. Note that: (i) the cost c^U_{MHSR} of a MHSR planning with any UBTD is an upper bound to c^*_{MHSR}, (ii) the optimal cost c^L_{MHDR} of an MHDR planning with any LBTD is a lower bound to c^*_{MHDR}. Consequently, the gap c^U_{MHSR}-c^L_{MHDR} is an upper bound to the gap c^*_{MHSR} -c^*_{MHDR}.

Fig. 6 plots the results for the case of study. We show the bound c^U_{MHSR}-c^L_{MHDR} for different values of K. Naturally, higher reduction sizes K yield tighter estimations of the c^*_{MHSR}-c^*_{MHDR} cost gap. The most accurate results we obtained ($K = 30$) showed that the maximum benefit achievable from traffic rerouting is lower than 8% in all the load conditions.

VIII. Conclusions

In this paper, we investigate the application of the concept of domination between sets of traffic matrices to the multi-hour network design. Both the static and dynamic routing cases are studied, which make use of the *total domination* and *domination* relation respectively. We propose algorithms to calculate simplified traffic series that either (totally) dominate or are (totally) dominated by the original MH traffic demand. The *dominating* MH demands (UBTD) can be used to design the network. We propose the *dominated* series (LBTD) as a tool to derive lower bounds to the network cost. Also, we define the *excess MH traffic demand* (ETD) and show its applicability to bind the suboptimality that is being caused by planning the network using a simplified traffic demand, instead of the original one. Results are encouraging and show that the techniques are effective for MH network design, for both the static and dynamic routing cases.

The authors believe that the traffic domination relation can become an effective tool to be applied to other network problems that deal with variations in the traffic demand. The authors are exploring its application for secure network planning against traffic anomalies and for multi-period planning.

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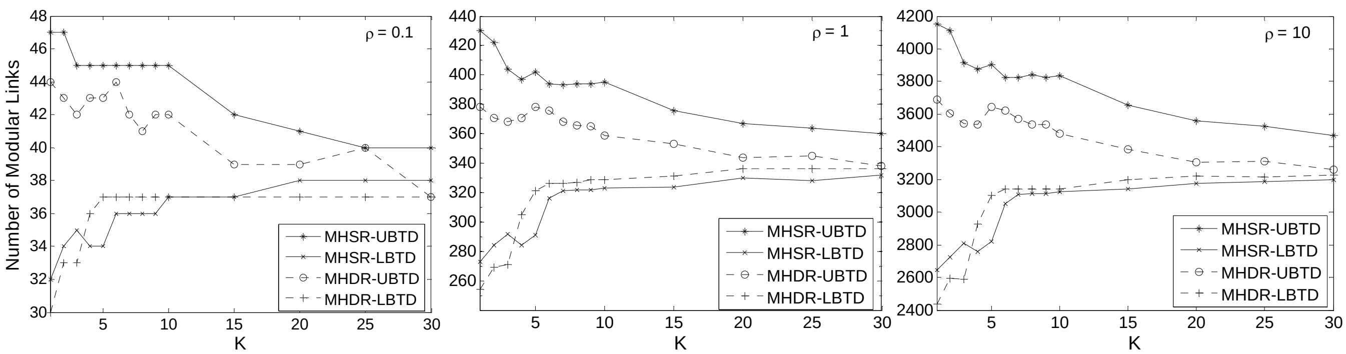
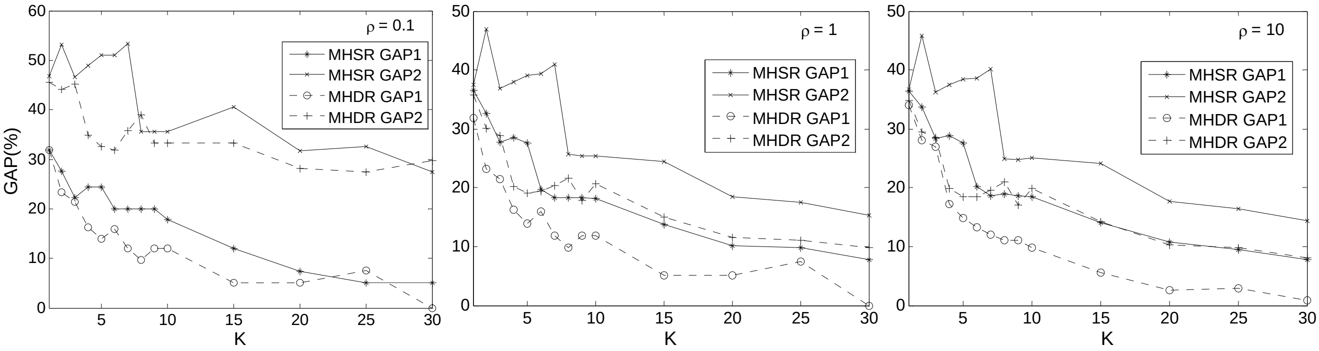


(a) (b) (c)

Fig. 4. Network cost obtained for the reduced LBTD and UBTD demands.



(a) (b) (c)

Fig. 5. Maximum suboptimality gaps Δc_1 and Δc_2 obtained from the LBTD and UBTD demands.

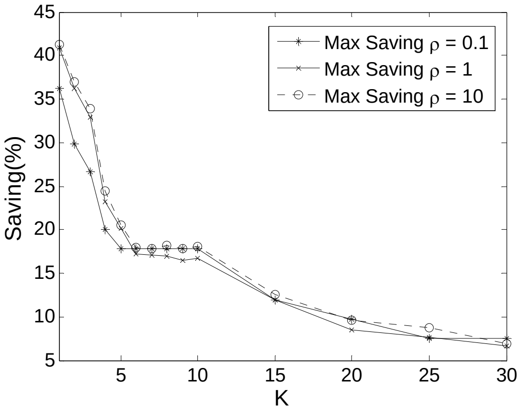


Fig. 6. Bound to the maximum cost saving obtained from traffic rerouting