Robust Upgrade in Optical Networks under Traffic Uncertainty

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Abstract— **Optical network operators face the challenge of upgrading the WDM network capacity to adapt to estimated traffic growths. Network upgrades are commonly carried out in scheduled intervals (i.e. every six months), using traffic forecasts. The uncertainty in the forecasts is a major issue in the capacity upgrading process. If it is not handled appropriately, the network is exposed to service degradation caused by an unexpected traffic progression. Despite of its relevance, the effects of uncertainty in the forecasts is a factor that has not been well studied in the literature. In this paper, we apply the robust optimization paradigm to incorporate this uncertainty into the network upgrade problem. Under this robust network upgrade model, we can dimension the network by tuning the tradeoff between network cost and robustness level. This proposal is applied to a case study where several experiments are conducted comparing different levels of robustness and different WDM technologies, namely pure 10G (single line rate), pure 40G SLR, pure 100G SLR and 10/40/100G MLR (mixed line rate).**

Index Terms—**Network upgrade, traffic growth, robust optimization, uncertainty modeling, optical WDM network, network planning.**

I. Introduction

THE ever-increasing traffic volumes in backbone networks poses a major challenge for telecommunication network operators. The traffic typically grows at figures as high as fifty percent per year [1] and the telecom revenue at eight percent per year [2]. Accommodating the traffic growth in a cost-effective fashion becomes a major issue for telecom network operators. To handle this problem, the usual traffic engineering (TE) techniques that aim to utilize the existing network resources optimally are not enough; and, periodic upgrades of capacity must be performed, scheduled to occur i.e. once or twice every year. These upgrade policies, referred to as Network Engineering (NE), look for the minimal cost investment in additional equipment, i.e., capital expenditure (CAPEX), guaranteeing that the network performance is met according to a traffic growth forecast.

From a simplified viewpoint, NE can be defined as “to put the bandwidth where the traffic is” (vs. TE that can be defined as “to put the traffic where the bandwidth is”) [3].

A major issue to consider in a network upgrade problem is the uncertainty associated with the traffic predictions. Network operators determine their investment in new equipment based on forecasts of future traffic demands. This new equipment should provide enough bandwidth to satisfy the Service Level Agreement (SLA) requirements of the operator. However, if the deviation of the forecasted traffic with respect to the real traffic in a future period is significant, the already-deployed investment may become insufficient. In this situation, traffic rejections and disruption in services can occur. Since network operators want to avoid such scenarios, understanding the traffic uncertainty is critical. However, this aspect has not been well studied for the network upgrade problem. Usually, a perfect knowledge of the future traffic is assumed skipping uncertainty considerations [4],[5],[6]. In other approaches, such as [7], the uncertainty is incorporated in the planning problem by means of stochastic programming where a finite number of scenarios and their associated probabilistic weights are considered in the objective function.

In this paper, we propose a new approach to introduce the uncertainty in the traffic forecasts into the network upgrade problem, by applying robust optimization [8],[9]. In robust optimization, the optimization process is not implemented over a deterministic traffic matrix but over an uncertainty set where all the realizable traffic matrices are confined. Thus, the design obtained must be suitable for any possible traffic instance in the uncertainty set. In contrast to [7], we can consider a continuum of values for the forecasted traffic matrices instead a few finite values and tune the tradeoff between robustness and overdimensioning cost.

This technique has been investigated in network routing problems where the network capacities are known [10]-[11]; and, in dimensioning and routing problems where both robust capacities and routings are found [12],[13], but without considering an upgrade scenario under traffic uncertainty. As far as the authors know, the present work is the first to apply a robust design technique to a network upgrade problem.

We will study our proposal in the framework of Wavelength division multiplexing (WDM) networks, the enabling technology in the backbone [14]. In WDM networks, traffic demands are routed on transparent all-optical

This work was partially supported by Spanish projects grants TEC2010-21405-C02-02/TCM (CALM) and TEC2010-12250-E (FIERRO), and also developed in the framework of "Programa de Ayudas a Grupos de Excelencia de la Región de Murcia” funded by F. Séneca.

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connections, called lightpaths. A lightpath is set up between a pair of transponders at two different ending nodes, occupying a single wavelength channel in each traversed link. Since the traffic carried onto a lightpath is not processed electronically at intermediate nodes, savings with respect to electronic switching equipment are achieved.

The particular set of lightpaths established over the physical topology constitutes the so-called virtual topology (VT). The planning problem, called virtual topology design (VTD), is a multilayer problem, where in the upper layer, electronic traffic demands (e.g. IP traffic) are routed on top of the lightpaths, whereas, in the lower layer, each lightpath is routed over the physical topology.

The rest of the paper is organized as follows. Section II introduces the deterministic Mixed-Integer Linear Programming (MILP) formulation of the VTD problem. In Section III, we provide the robust version of the previous formulation by applying a probabilistic model of the traffic uncertainty. Section IV, describes the case study and the obtained results. Finally, Section V concludes the paper.

II. Deterministic Model

This section details the deterministic model used to dimension the capacity in a WDM optical network. This model is a multilayer MILP formulation that expands the single-layer ILP proposed in [4] by considering electronic grooming of the higher layer traffic (e.g. IP traffic) onto the lightpaths. The cost of the optical equipment and the cost of the electronic switching are extracted from [15].

Let $G(N,E)$ be the graph of the network topology where N is the set of nodes and E is the set of unidirectional fiber links. We refer to the set of fiber links initiated or ending at node $n \in N$ as δ_n. We assume that a common wavelength grid is used in all the network links, where W is the set of wavelength channels. We denote as D the set of all traffic demands between two nodes, and $a(d)$ and $b(d)$ denote the initial and end nodes of demand $d \in D$, respectively. For each demand d, h_d represents the volume of the demand in bit rate units (Gbps) and $\mathbf{H}=\{h_d,\ d \in D\}$ is the traffic vector (or traffic matrix, we use both terms indistinctly in the paper) containing all the demand volumes.

The set R represents the available types of optical transponders. For each transceiver type $r \in R$, c_r denotes its bit rate in Gbps. The set Δ refers to the fiber nodal degrees supported for the available optical switching equipments.

We denote as P the set of paths p that are candidates to support a lightpath. The set of paths traversing fiber $e \in E$ is denoted as P_e. Finally, the set of paths initiated at node $i \in N$ and ending at node $j \in N$ is denoted as P_{ij}.

The decision variables of the problem are:

- $x^d_{ij} \in [0,1]$. Fraction of the electronic traffic volume associated to demand $d \in D$ carried onto the bundle of lightpaths established between nodes i and j.
- $y_{pr} = \{0,1,2,..\}$ Number of lightpaths established on $p \in P$ using transceiver type $r \in R$.
- $f_e = \{0,1,2,..\}$. Number of fibers in service on link $e \in E$.
- $s_{nk} = \{0,1\}$. s_{nk} takes the value 1 if an optical switching device of degree k is installed at node $n \in N$.

Then, the problem can be formulated as:

$$\text{Min} \left\{ \begin{array}{l} \sum_{p \in P, r \in R} \left(2 \cdot \tau_r + NR_{pr} \rho_r\right) y_{pr} + \\ + \sum_{e \in E} \phi_e \cdot f_e + \sum_{n \in N, k \in \Delta} \varsigma_k \cdot s_{nk} + \sum_{d \in D, (i,j) \in N \times N} \varepsilon \cdot h_d x^d_{ij} \end{array} \right\} \quad (1a)$$

Subject to

$$\sum_{j \in N} \left(x^d_{ij} - x^d_{ji}\right) = \begin{cases} 1 & \text{if i} = a(d) \\ -1 & \text{if i} = b(d),\ \text{i} \in N, d \in D \\ 0 & \text{otherwise} \end{cases} \quad (1b)$$

$$\sum_{d \in D} h_d x^d_{ij} \le \sum_{p \in P_{ij}, r \in R} c_r \cdot y_{pr}, \quad (i,j) \in N \times N \quad (1c)$$

$$\sum_{p \in P_e, r \in R} y_{pr} \le |W| \cdot f_e, \quad e \in E \quad (1d)$$

$$\sum_{e \in \delta_n} f_e \le \sum_{k \in \Delta} k \cdot s_{nk}, \quad n \in N \quad (1e)$$

$$\sum_{k \in \Delta} s_{nk} \le 1, \quad n \in N \quad (1f)$$

The objective function (1a) minimizes the total network cost, considering the following cost coefficients:

- τ_r: Cost of an optical transponder at bit rate $r \in R$.
- ρ_r: Cost of an optical 3R regenerator at bit rate $r \in R$.
- N_{pr}: Number of optical 3R regenerators at bit rate $r \in R$ required to accomplish a transparent path $p \in P$.
- φ_e: Cost of a fiber in link $e \in E$, combining the Optical Line Amplifier (OLAs), the Dispersion Compensating Fibers (DCFs), and the Dynamic Gain Equalizers (DGEs) costs.
- ζ_k: Cost of an optical switch (such as, Optical Cross Connect, OXC) with degree k.
- ε: Electronic Switching Cost per 1 Gbps.

The flow conservation constraints (1b) ensure that all the traffic volume of demands $d \in D$ is carried. The lightpath capacity constraints (1c) guarantee that traffic allocated in a lightpath does not exceed the bit rate of the transceivers. The fiber link capacity constraints (1d) ensure that the number of fibers is sufficient to carry the lightpaths routed on the link. The nodal capacity constraints (1e) force to install optical switching equipment at the nodes with enough fibre nodal degree. The fact that one unique optical switch can be installed in a node is considered in (1f).

III. Robust Model under Probabilistic Uncertainty Set

In this section, we present a robust dimensioning approach suitable for network upgrade problems. The key idea of this approach is to represent the uncertainty set by means of the same probabilistic model used to compute the traffic forecasts. First, we explain this probabilistic uncertainty set. Then, we construct the robust problem by introducing the uncertainty set in the deterministic problem presented in Section II.

A. Traffic Uncertainty Set

Network operators employ forecasts of future traffic to dimension the network resources. Normal probability distributions are commonly applied to estimate these forecasts [16]. This assumption is based on the central limit theorem, since the traffic offered to the backbone networks grooms many different services from many low-bandwidth sources. In this paper, we will follow this basic assumption.

If we consider that future volume demand h_d^*, $d\in D$ can be forecasted according to a normal distribution $N(\mu_d, \sigma_d)$, where μ_d is the expected volume and σ_d represents the uncertainty in the forecast, and the correlation coefficients $\rho_{dd'}$ between the demands $(d,d')\in D\text{x}D$ are known, then the traffic vector $\mathbf{H}$ can be estimated by a multivariate normal distribution $MVN(\boldsymbol{\mu},\boldsymbol{\Sigma})$, such that the mean vector $\boldsymbol{\mu}$ and the covariance matrix $\boldsymbol{\Sigma}$ are computed by (2) and (3), respectively:

$$\boldsymbol{\mu} = \{\mu_d , d \in D\} \tag{2}$$

$$\boldsymbol{\Sigma}=\begin{pmatrix} \sigma_{d_1}^2 & \rho_{d_1d_2}\cdot\sigma_{d_1}\cdot\sigma_{d_2} & \cdots & \rho_{d_1d_{|D|}}\cdot\sigma_{d_1}\cdot\sigma_{d_{|D|}} \\ \rho_{d_1d_2}\cdot\sigma_{d_1}\cdot\sigma_{d_2} & \sigma_{d_2}^2 & \cdots & \rho_{d_2d_{|D|}}\cdot\sigma_{d_2}\cdot\sigma_{d_{|D|}} \\ \vdots & \vdots & \ddots & \vdots \\ \rho_{d_1d_{|D|}}\cdot\sigma_{d_1}\cdot\sigma_{d_{|D|}} & \rho_{d_2d_{|D|}}\cdot\sigma_{d_2}\cdot\sigma_{d_{|D|}} & \cdots & \sigma_{d_{|D|}}^2 \end{pmatrix},\ \{d_1,d_2,\cdots,d_{|D|}\}\in D \tag{3}$$

Under this MVN model, the tolerance region, i.e., the region containing exactly $100\cdot P$ % of the possible matrices [17], is the ellipsoidal region defined by:

$$\Theta(P)=\left\{\mathbf{H}^*\in\Re_+^{|D|} : (\mathbf{H}^*-\boldsymbol{\mu})^{\mathrm{T}}\boldsymbol{\Sigma}^{-1}(\mathbf{H}^*-\boldsymbol{\mu})\le\chi^2(1-P,|D|)\right\} \tag{4}$$

where $\chi^2(1\text{-}P,|D|)$ is the value in $\chi^2(|D|)$, the Chi-Square distribution with $|D|$ degrees of freedom, above the $100\cdot P$ % of realizations. In the robust model in this paper, this tolerance ellipsoid $\Theta(P)$ for a given P will be our traffic uncertainty set. Then, since the robustness of the model can be controlled by changing the value of this P parameter, we refer it as *robustness level*.

B. Robust Dimensioning Model

The robust formulation (5) computes the minimum cost solution that is able to handle any traffic vector $\mathbf{H}^*$ contained in the ellipsoid (4). Then, it guarantees that the capacity upgrade is sufficient with at least a P % probability. The model only differs from (1) in the objective function (1a) and the lightpath capacity constraints (1c).

$$\text{Min}\left\{\begin{aligned}&\sum_{p\in P,r\in R}\left(2\cdot\tau_r + NR_{pr}\rho_r\right)y_{pr} + \sum_{e\in E}\phi_e\cdot f_e + \\ &+\sum_{n\in N,k\in\Delta}\varsigma_k\cdot s_{nk} + \sum_{(i,j)\in N\times N}\max_{\mathbf{H}^*\in\Theta(P)}\left\{\sum_{d\in D}h_d^* x_{ij}^d\right\}\end{aligned}\right\} \tag{5a}$$

Subject to

(1b), (1d), (1e), (1f)

$$\max_{\mathbf{H}^*\in\Theta(P)}\left\{\sum_{d\in D}h_d^* x_{ij}^d\right\}\le\sum_{p\in P_{ij},r\in R}c_r\cdot y_{pr},\quad (i,j)\in N\times N \tag{5b}$$

The rest of this section is targeted to show an equivalent formulation to (5), that appropriately handles the expression:

$$\max_{\mathbf{H}^*\in\Theta(P)}\left\{\sum_{d\in D}h_d^* x_{ij}^d\right\} \tag{6}$$

First, we note that for each $(i,j)\in N\text{x}N$ and a IP fixed-flow routing vector $\mathbf{x_{ij}} = \{x^d_{ij} : d\in D\}$ satisfying (1b), the problem (6) can be formulated in vector notation as:

$$\max_{\mathbf{H}^*}\{\mathbf{H}^*\cdot\mathbf{x_{ij}}\},\quad \text{subject to:} \tag{7a}$$

$$(\mathbf{H}^*-\boldsymbol{\mu})^{\mathrm{T}}\boldsymbol{\Sigma}^{-1}(\mathbf{H}^*-\boldsymbol{\mu})\le\chi^2(1-P,|D|) \tag{7b}$$

The ellipsoidal region can be represented as:

$$\mathbf{H}^*=\boldsymbol{\mu}+\mathbf{A}\cdot\mathbf{u},\quad \|\mathbf{u}\|\le 1,\quad \mathbf{A}=\left(\chi^2(1-P,|D|)\cdot\boldsymbol{\Sigma}\right)^{1/2} \tag{8}$$

Then, the problem (7) is equivalent to:

$$\max_{\mathbf{u}}\{\boldsymbol{\mu}^{\mathrm{T}}\cdot\mathbf{x_{ij}}+[\mathbf{A}\cdot\mathbf{u}]^{\mathrm{T}}\cdot\mathbf{x_{ij}}\},\ \text{subject to:} \tag{9a}$$

$$\|\mathbf{u}\|\le 1 \tag{9b}$$

This formulation can be simplified by removing the constant term $\boldsymbol{\mu}^{\mathrm{T}}\cdot\mathbf{x_{ij}}$ from the objective function and transforming the second term introducing the vector $\boldsymbol{\alpha}_{\mathbf{ij}} = \{\alpha_d=\mathbf{A_d}\cdot\mathbf{x_{ij}};\ d\in D\}$, where $\mathbf{A_d}$ is the *d-th* row of the matrix $\mathbf{A}$.

$$\max_{\mathbf{u}}\{\boldsymbol{\alpha}_{\mathbf{ij}}^{\mathrm{T}}\cdot\mathbf{u}\},\ \text{subject to:} \tag{10a}$$

$$\mathbf{u}^{\mathrm{T}}\cdot\mathbf{u}\le 1 \tag{10b}$$

The above formulation is modeling a convex problem that can be solved by applying the Karush-Kuhn-Tucker (KKT) optimality conditions:

$$L(\mathbf{u},\lambda) = \boldsymbol{\alpha}_{\mathbf{ij}}^{\mathbf{T}} \cdot \mathbf{u} + \lambda \cdot (1 - \mathbf{u}^{\mathbf{T}} \cdot \mathbf{u});$$
$$\nabla_{\mathbf{u}} L = \boldsymbol{\alpha}_{\mathbf{ij}}^{\mathbf{T}} - 2\lambda \mathbf{u} = \mathbf{0} \quad (11a)$$

$$\mathbf{u}^{\mathbf{T}} \cdot \mathbf{u} \le 1 \quad (11b)$$

$$\lambda \ge 0 \quad (11c)$$

$$\lambda \cdot (1 - \mathbf{u}^{\mathbf{T}} \cdot \mathbf{u}) = 0 \quad (11d)$$

The solution to the KKT conditions are that $\lambda = \|\boldsymbol{\alpha}_{\mathbf{ij}}\|/2$, $\mathbf{u} = (1/2\lambda)\cdot\boldsymbol{\alpha}_{\mathbf{ij}}$, so that $\|\mathbf{u}\|=1$ and the maximal value of (10) becomes $\|\boldsymbol{\alpha}_{\mathbf{ij}}\|$.

Incorporating these results to the robust model (5), we have:

$$\text{Min} \left\{ \begin{array}{l} \sum_{p\in P, r\in R} \left(2\cdot\tau_r + NR_{pr}\,\rho_r\right) y_{pr} + \sum_{e\in E} \phi_e \cdot f_e + \\ + \sum_{n\in N, k\in\Delta} \varsigma_k \cdot s_{nk} + \sum_{(i,j)\in N\times N} \left\{ \sum_{d\in D} \mu_d x_{ij}^d + \left\|\boldsymbol{\alpha}_{\mathbf{ij}}\right\| \right\} \end{array} \right\} \quad (12a)$$

Subject to

(1b), (1d), (1e), (1f)

$$\sum_{d\in D} \mu_d x_{ij}^d + \left\|\boldsymbol{\alpha}_{\mathbf{ij}}\right\| \le \sum_{p\in P_{ij}, r\in R} c_r \cdot y_{pr}, \quad (i,j)\in N\times N \quad (12b)$$

Note that the left-hand side in (12b) is the maximal traffic load at a bundle of lightpaths between nodes i and j under any traffic vector $\mathbf{H^*}$ contained in the ellipsoid (4). This term is composed of two terms representing:

- $\sum_{d\in D} \mu_d x_{ij}^d$: load associated with expected volumes μ_d.
- $\|\boldsymbol{\alpha}_{\mathbf{ij}}\|$, where $\boldsymbol{\alpha}_{\mathbf{ij}} = \{\alpha_d = \mathbf{A_d}\cdot\mathbf{x_{ij}};\ d\in D\}$: load associated with the uncertainty on the estimation of the expected volumes μ_d.

Therefore, given an IP fixed-flow routing vector $\mathbf{x}$, we have a simple formula to compute the lightpath load (l_{ij}) that we must consider for robust provisioning of the optical network under our uncertainty model:

$$l_{ij} = \sum_{d\in D} \mu_d x_{ij}^d + \left\|\boldsymbol{\alpha}_{\mathbf{ij}}\right\|, \quad (i,j)\in N\times N \quad (12)$$

IV. Robust Network Upgrade Case Study

A. Network Upgrade Scenario

We first study a single period robust network upgrade scenario from the perspective of a Network Operator (NO), which owns the optical infrastructure. The objective of the NO is to determine the investments in additional optical equipment to carry the forecasted demand over the next planning period (e.g. the next six or twelve months). The NO is interested in performing the upgrade meeting the following constraints:

1. The capacity upgrade is accumulative to amortize the legacy equipment: the upgrade consists of adding new IP and optical equipment without removing the existing one.
2. The existing IP routing over the lightpaths is kept fixed in the upgrade, to minimize disruptions on existing services. IP routing stability, even when the number of lightpaths between two nodes increases, is possible thanks to techniques like lightpath bundling [18].
3. We seek for a robust design that fully carries the traffic increases occurring until the next network upgrade, with some probability P.

Then, we follow the methodology outlined below:

1. We find an initial network design at period t_0, by solving the formulation (1) for the current traffic matrix. This network design provides an IP routing $\mathbf{x}$ onto the virtual topology; and, the legacy optical equipment (transponders, 3R regenerator, OLAs, …)
2. For the planning period $t_1 > t_0$ and for each node pair (i,j), we compute the robust load (l_{ij}) at each "bundle" of lightpaths by replacing $\mathbf{x}$ in the formula (12). Then, we compute the *additional* optical equipment by solving the formulation (13) derived from (1):

$$\text{Min} \left\{ \sum_{p\in P, r\in R} \left(2\cdot\tau_r + NR_{pr}\rho_r\right) y_{pr} + \sum_{e\in E} \phi_e \cdot f_e + \sum_{n\in N, k\in\Delta} \varsigma_k \cdot s_{nk} \right\} \quad (13a)$$

Subject to

(1d), (1e), (1f)

$$l_{ij} \le \sum_{p\in P_{ij}, r\in R} c_r \cdot y_{pr}, \quad (i,j)\in N\times N \quad (13b)$$

$$y_{pr} \ge y_{pr}', p\in P, r\in R \quad (13c)$$

$$f_e \ge f_e', e\in E \quad (13d)$$

$$s_{nk} = 0, n\in N, k < \arg_k \{ s_{nk}' = 1 \} \quad (13e)$$

In the constraints (13c)-(13e), y_{pr}', f_e' and s_{nk}' denote the legacy optical equipment at t_0. These constraints force the capacity upgrade to be accumulative.

Concerning the computation complexity of this methodology, we must note that load computation by (12) is trivial; and, formulations (1) and (13) can be replaced for efficient heuristics to solve large-sized problem instances.

B. Study case

We will apply the methodology detailed in the previous subsection to the US long-haul network Intenet2 (|N|=9) [19], using 80 wavelengths per fiber and a grid spacing of 50 GHz. We will consider different optical WDM technologies based

TABLE I
TRANSPONDERS PARAMETER

Bit Rate (Gbps)	Modulation Forma	Transparent Reach (Km)	Cost of one transponder (τ_r)
10	OOK	3000	1
40	BPSK	1600	3
100	PDM-QPSK	800	6

on the transponder bit rate, namely pure 10G SLR (single line rate), pure 40G SLR, pure 100G SLR and 10/40/100G MLR (mixed line rate)

The different network costs are modeled as follows:

- $\varepsilon = 1.2$. We estimate the electronics cost as the cost of the electronic IP equipment installed at a fully equipped node working at maximum throughput traffic [15].
- τ_r: As in Table I extracted from [20].
- ρ_r: 140% of the equivalent transponder cost (as in [15]).
- φ_e: From [15].
- ζ_k: From [15].

For each transponder type r, transparent reaches (maximal length of a lightpath without requiring a 3R regenerator) and modulation formats are shown in Table I ([20]). The number of 3R regenerators N_{pr} is precomputed for each p using the transparent reach corresponding to r. All the optical equipment costs extracted from [15] corresponds to 80 wavelengths and ultra-long-haul equipment (3000 km).

The traffic matrix at t_0, known without any uncertainty, is the same as in work [19] and is normalized to 3 Tbps. We use the assumptions below to characterize the traffic growth:

1. The total volume of the average traffic matrix $\boldsymbol{\mu}$ grows exponentially a fifty percent per period (e.g., a year) [1].
2. In a given planning period $t_1 > t_0$, all the demands $d\in D$, have the same coefficient of variation (CV_d) i.e., the same ratio between the average traffic volume μ_d and the standard deviation σ_d in the traffic forecast.
3. The CV_d grows exponentially with the planning period reflecting that the uncertainty increases with the time.
4. The correlation coefficients $\rho_{dd'}$ between the demands $(d,d')\in DxD$ are considered negligible.

C. Numerical Results

1) Cost vs. Planning Period

In this experiment, we study the robust network upgrade along four consecutive planning periods of six months from the present period t_0. CV will follow the above mentioned exponential law in the range of values used in [16], taking these figures {0, 0.0225, 0.05, 0.0837, 0.125} in each period. We repeat the experiment for each WDM technology using a $P = 0.999$. The rest of the parameters of the experiments take the values indicated in Section IV.C.

In Fig. 1, the costs obtained in the different planning periods are shown. Logically, the 10/40/100G MLR design obtains the lowest costs, since it is the most flexible approach. The 100G case provides the worst figures due to its short transparent reach (800 Km) what forces to use more intensively the optical equipment depending on the number of

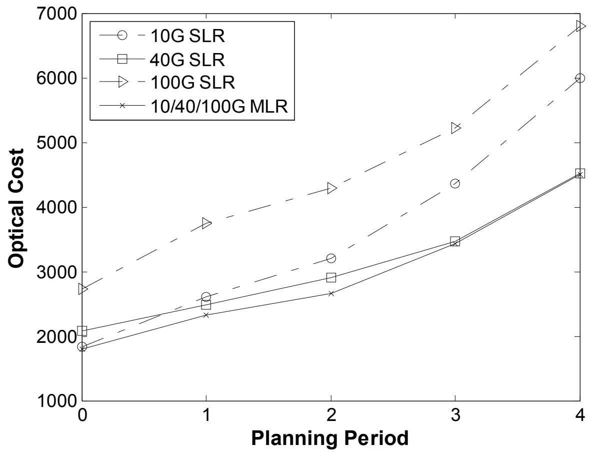

Fig. 1. Overall Optical cost vs. Planning Period.

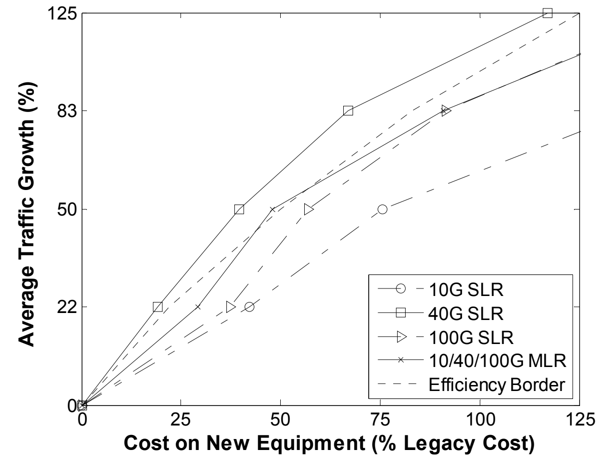

Fig. 2. Traffic Growth vs. Cost of New Equipment.

lightpaths: transponders and 3R regenerators. The 10G results improve the previous ones as the longer transparent reach reduces the usage of transponders and 3R regenerators. However, the lower bit rate per wavelength leads to increase the number of fibers lit, and thus higher costs in OLAs, DCFs, DGEs and OXCs. Conversely, the performance of the 40G SLR designs is almost equivalent to the MLR case since the larger lightpath capacity and the volume discounts associated to these larger bit rates allow a good balance between the usage of lighpath-associated and fiber-related equipments.

Figure 2 shows the same data as in Fig. 1, but they are "reformatted" to provide an interesting insight in the performance of the network upgrade depending on the WDM technology. In Fig. 2, the optical costs are represented in the x-axis as cost increments with respect to the cost of the legacy infrastructure, i.e., the cost of the optical equipment currently installed at t_0. The planning periods are replaced by the corresponding average traffic volume growth and shown in the y-axis. Now, we can see clearly the relation between the investment in new equipment and the traffic growth that this investment can support. We also show the line $y=x$ as a dotted line separating the plot in two areas: (a) above the line, the ratio between the traffic increment and the cost increment is larger than one; and, (b) below the line, the ratio is less than one. We will call this line as "the efficiency border" of the network upgrade. The NO will attempt that growths in traffic can be handled with lower growths in the cost infrastructure.

From Fig. 2, it is clear that the 40G SLR design outperforms the other ones, and it is above the efficiency border in the range of cost increment from 0 to 125%).

2) Impact of the value of P on the Cost

Now, we repeat the previous experiment for different values of P = {0.001, 0.1, 0.25, 0.5, 0.75, 0.9, 0.999}. In Fig. 3, we show the results by averaging the optical equipment costs over the planning periods and normalizing with the cost found for the $P = 0.5$ case. We observe that, for all the WDM technologies, the extra upgrading cost that comes from introducing the robustness level P is fairly moderate even for P levels as high as 0.999.

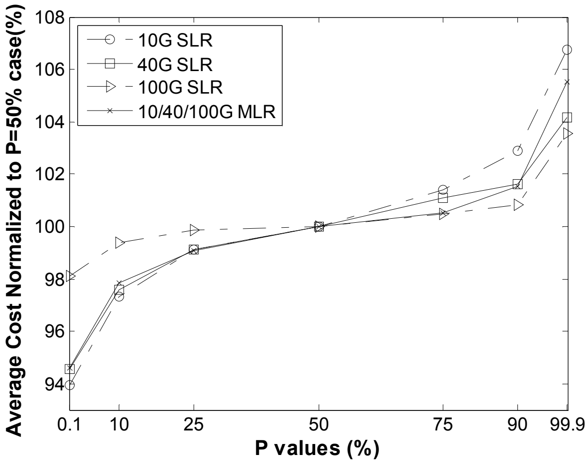

Fig. 3. Cost vs. P Values.

V. Conclusion

In this paper, we propose a robust optimization approach to incorporate into the network upgrade problem the inherent uncertainty in the traffic forecast. We model the traffic forecast as a multivariate normal distribution, which is able to flexibly capture the uncertainty in the traffic growth. Then, we present the robustness model that permits designing the network, guaranteeing that the capacity upgrade can support the traffic growth with a P % probability (called robustness level).

The robust model is applied to an upgrade problem in optical WDM networks, where the network operator needs to determine the CAPEX to invest in its network periodically (i.e. every six months) to support future traffic growths. This problem is studied for different WDM technologies: 10G SLR, 40G SLR, 100G SLR, and 10/40/100G MLR; and, also for different robustness levels P. We find that MLR and 40G SLR provide the lowest cost designs and the most "efficient" upgrades, namely supporting more traffic growth with less additional investment. The study of the evolution of the upgrade cost when we vary the value of P shows that very high robustness levels (i.e. P=0.999) can be obtained with little over-upgrading with respect to very low robustness levels (i.e. P=0.1).

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