Optimum maneuvering under time constraints for high speed vehicles

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***Abstract*—Nowadays, research on Vehicular Technology aims at automating every single mechanical element of vehicles, in order to increase safety for passengers, reduce human driving intervention and provide them with entertainment services on board. Automatic trajectory tracing for vehicles under specially risky circumstances is a field of research which is currently gaining a lot of attention. In this article we show some results on how to develop useful policies to change the heading of a vehicle at high speeds with the mathematical optimization of some already established mobility conditions for the car. Different strategies are presented as well as a discussion on how communications can be used to implement these schemes.**

I. INTRODUCTION AND MOTIVATION

According to statistics of the DGT (*Spanish National Traffic Administration*), around 91% of fatal traffic accidents take place on highways and conventional roads [1] in Spain. 71% of these accidents occur due to the collision of two or more vehicles in transit, against a suddenly appearing obstacle on the road, or because of pedestrians or animals running-over. This information shows how important is to develop new technologies which can assist the driver or make autonomous decisions to increase passengers' safety.

In this regard, the automotive industry is currently moving towards the automation of every single aspect of the driving experience [2]. The main trend, before cars are fully automated, consists of improving the driver assistance by providing new mechanisms which can increase safety and comfort. This can be effectively implemented by the installation of devices which can monitor the environment continuously and, at the same time, determine under which situations the driver needs some kind of specific aid (through somewhat advanced smart processing applications). By monitoring the environment we mean using specific cameras or sensors which can acquire useful information to improve the driving experience.

Communications will also play an important role when distributing information monitored by sensors on-board vehicles to enhance the safety conditions of passengers while driving. The standard WAVE 1609/802.11p [3] has been designed for this purpose, and it covers the general communication schemes which vehicles will implement in the future to support V2V (*vehicle to vehicle*) and V2I (*vehicle to infrastructure*) communications. Through the use of communications the amount of information which a car processes is much higher, and thus, with a proper processing scheme, it can be really useful to make timely and smarter decisions which, when isolated, would not be possible.

In virtue of this information, we can conclude that by using such technologies vehicles could benefit from the environmentally collected data to make autonomous decisions in order to reduce the probability of a fatal event (vehicle-vehicle collision, pedestrian-animal running-over) when driving at high speeds. In the present study we focus on the development of useful procedures to dynamically optimize the trajectory of a vehicle under timing constraints, as regards changing the lateral position on the road to maximize the lateral distance within an interval of t_f s, and simultaneously optimize some other mobility parameters like the total acceleration along the path (to reduce the impact of sudden inertial changes). Since this mathematical optimization framework will be extended for cooperative collision avoidance (CCA) in a future work, we will analyze subtly how they can be implemented with our proposal.

In summary, the main contributions of this paper are:

1) An easy way to implement a procedure to obtain real-time maneuvers for a high-speed vehicle subject to timing constraints (the maneuver must be performed in a maximum interval time of t_f s).
2) To the best of the authors' knowledge, no previous work has dealt with the mathematical optimization of both safety and driving comfort for automatic trajectory tracing.
3) A mathematical framework to analyze more general scenarios for the CCA policies.

This paper is organized as follows. In Section II the reader will find some previous related work which served as a reference for the development of our proposal. Section III will state the problem's most important features and different illustrative results to evaluate the performance of our trajectory tracing algorithm. Section IV will conclude the paper with some important remarks and pointing out the future work derived from the present investigation.

II. RELATED WORK

Among the previous studies regarding active steering on highways, we can mention a few ones which awake our interest. That is the case of [4], where we can find a mathematical evaluation in which some interesting properties of active maneuvering are obtained by applying *Optimal Control theory* (OC) to the calculation of optimum trajectories for a specific final heading. Either in minimum time or in minimum distance, results in this paper show that trajectory tracing for just one vehicle when looking for a specific heading can be useful when dealing with general robots, but not as much when talking about vehicles, which essentially would expect to avoid an accident, regardless of the final heading. In [5] we can find an approach to solve the problem of tracing the trajectory for a car between two fixed points. This type of problem is a BVP (*Boundary Value Problem*) in which the main concern is to study the trajectory between two fixed points while optimizing the time employed to complete the trajectory according to some physical constraints in terms of maximum acceleration. This evaluation gives a useful hint on the analysis of OC problems by providing a mathematical functional based on nearly-time optimality for the solution of the problem. However, formulating this kind of problems this way entails suffering from convergence issues as well as constraints in values for mobility parameters (such as speed). Furthermore, fixed final boundaries are rarely met in real circumstances due to the obvious variability of the environmental conditions. Another approach for the establishment of an optimal control law for a pair of Dubin's vehicles (cars moving characterized by a sinusoidal differential equations model) is found in [6], where authors mathematically obtain trajectories which transfer the two Dubin's vehicles from pre-established initial positions to fixed final locations in space. The optimization targets in [6] include the minimization of acceleration, speed and distance along the trajectory and the maximization of the distance between the two vehicles during the maneuver. It is also a BVP where here we can find some sort of cooperative trajectory optimization. On the other hand, the proposal in [7] describes the collision avoidance problem as an IVP (*Initial Value Problem*) where a vehicle divides the trajectory tracing procedure into three stages (each one with its own functional). The first one consists of steering to simply avoid the collision against the obstacle; the second phase affords the reorientation of the trajectory to avoid colliding against the lateral protections; and the last one consists of relocating the car in the same lane it was circulating. Moreover, this paper provides a realistic mobility model of the vehicle, in which there is a full mechanical analysis of the vehicle's response. We find thus here the best approach to the specific topic we are exploring in this evaluation, but the specific mechanical model is very complex, something we can avoid with our proposal.

III. PROBLEM STATEMENT AND RESULTS

In this section we will at first give a description of the specific problem we tackle in this evaluation, introducing the mathematical equations needed to solve the problem and the tools used in this regard. We will also analyze the performance of the trajectory tracing procedure along with its qualitative connection to vehicular networking for cooperative collision avoidance (CCA).

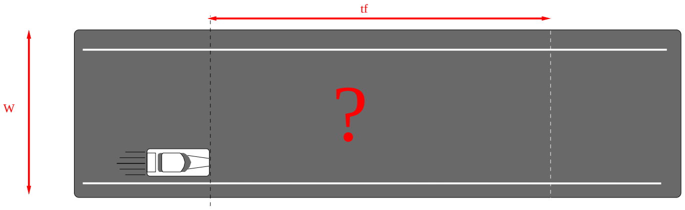

Fig. 1. Maximization of lateral distance after t_f

A. Scenario description and formulation

The general problem relies on coming up with a trajectory tracing procedure to deal with critical situations in which a vehicle has to avoid the collision against a suddenly appearing obstacle (another vehicle, a pedestrian or an animal) in the middle of the road.

However, in this paper we will concentrate on the simplest version of the problem in order to build a profound understanding on how trajectories for a single vehicle evolve according to the specific optimization requirements we set, and how the behavioral restrictions to which the car's mobility is constrained affect the maneuver. In this so straightforward version of the problem, we assume there is only one vehicle circulating on a W-meter-wide road (regardless of the number of lanes). This vehicle has to trace a trajectory in an interval of t_f seconds after the maneuver has begun (see Figure 1). During the trajectory as well as after t_f seconds the vehicle will try to adjust its mobility evolution as much as possible to the specific optimization requirements which are previously set in the mathematical calculations. In this particular case we will refer to the maximization of the lateral distance to the lateral boundaries of the road, while minimizing the sum of the square of the instantaneous acceleration along the trajectory, all within the interval of t_f seconds configured beforehand. Maximizing the lateral distance implies in this case leaving the maximum possible road width on both sides of the vehicle, thus minimizing the probability of colliding with the lateral protections. On the other hand, by simultaneously minimizing the sum of the square of the instantaneous accelerations we can change the general appearance of the trajectory in such a way that the inertial impact on mobility of passengers is reduced.

For the analysis of the problem we first need to consider the mathematical model of the vehicle, which is represented by the system of differential equations:

$$\begin{cases} \dot{x}_1(t) = v_1(t) \\ \dot{v}_1(t) = a_1(t) \end{cases} \tag{1}$$

$x_1(t)$ refers to the position of vehicle 1 in time t, $v_1(t)$ denotes the current speed of vehicle 1, and $a_1(t)$ defines the acceleration of vehicle 1 along time. We take $a_1(t)$ as the control variable which we have to manipulate in real time in order to handle the trajectory traced by the vehicle. We assume these mobility equations to govern only the lateral displacement of the vehicle (we consider the longitudinal speed to be constant and fixed to 120 km/h during all the trajectory).

Furthermore, car's mobility has to respect the physical limits imposed by the inertial laws of kinematics. This means that the vehicle can only turn at a maximum established acceleration, which depends on the longitudinal speed (the higher the speed, the harder the maneuver). Considering this, we can establish the mechanical restrictions of the problem at hand:

1) **Lateral acceleration restrictions**. The absolute value of the lateral acceleration cannot take a value higher than the limit $c(v_i)$ m/s^2, where v_i is the constant longitudinal speed of the vehicle and $c(\cdot)$ is a function of the longitudinal speed.

$$|a(t)| \leq c\left(v_i\right) \tag{2}$$

2) **Lateral position restrictions**. The vehicle can only have a lateral displacement inside the width limits of the road.

$$0 < x_1\left(t\right) < W \tag{3}$$

The third and most important aspect of the statement of the problem is the functional we want to optimize. Although it is possible to formulate very different kinds of functionals according to the specific target we want to optimize, we will focus on just two main objectives: minimization of the variance of the lateral distance, and minimization of the sum of the instantaneous accelerations along the trajectory. This can be justified due to the fact that the lower the variance of the distances of the lateral gaps (between two vehicles, two obstacles or vehicle to obstacle), the highest the lateral distance to other elements on the road. On the other hand, minimizing the square of the acceleration along the trajectory will turn into a null lateral inertia when approaching the end of the path as we will see later.

The four proposed functionals are shown next:

1) **Final lateral distance maximization and square of instantaneous acceleration minimization**. In this case we want to minimize the final variance of the lateral distances left by the vehicle when reaching the final position, while minimizing the square of the acceleration during the whole trajectory. The equation corresponding to the functional takes the form:

$$J_{D1} = x_1^2\left(t_f\right) + (W - x_1(t_f))^2 + \int_0^{t_f} a_1^2(t)dt \tag{4}$$

2) **Final lateral distance maximization**. In this case we skip the optimization of the square of the instantaneous acceleration along the trajectory, while maximizing the final lateral distance.

$$J_{D2} = x_1^2\left(t_f\right) + (W - x_1(t_f))^2 \tag{5}$$

3) **Instantaneous lateral distance maximization and square of instantaneous acceleration minimization**. This functional aims at maximizing the instantaneous lateral distance, while minimizing the square of the instantaneous acceleration along the trajectory.

$$J_{D3} = \int_0^{t_f} \left[x_1^2\left(t\right) + \left(W - x_1\left(t\right)\right)^2 + a_1^2\left(t\right)\right] dt \tag{6}$$

4) **Instantaneous lateral distance maximization**. In this case we skip the optimization of the square of the instantaneous acceleration along the trajectory, while maximizing the instantaneous lateral distance.

$$J_{D4} = \int_0^{t_f} x_1^2\left(t\right) + \left(W - x_1\left(t\right)\right)^2 dt \tag{7}$$

For the optimization of the aforementioned functionals we will rely on the *Gradient Projection Algorithm*, some sort of *Gradient Descent* procedure which includes functionals which can only be optimized inside a region determined by the constraints of the problem [8]. This method requires to discretize the trajectory in *N* stages so that the path optimization can be performed computationally. Apart from the evaluation of trajectories, in the next subsections we will explore how the discretization factor *N* influences in the determination of trajectories and the resolution we need according to the specific functional in order to reach a reasonable trade-off between curve error and computational weight.

B. Final lateral distance maximization

In this first subsection we compare the performance of functionals J_{D1} and J_{D2} for the configuration of parameters in Table I, where the instantaneous lateral distance of the vehicle along the trajectory is not maximized, but the final (at t_f s) lateral distance to the lateral protections otherwise. If we obtain the mobility evolution (acceleration, speed and position evolution along the trajectory) for two different values of t_f (2 and 10 seconds), we can notice from Figures 2 and 3 that for longer time intervals to execute the maneuver vehicles do not need to reach the maximum acceleration stated by the model. On the other hand, for lower values of t_f the steering maneuver needs to use the maximum allowable values of the lateral acceleration to maximize the functional at the end of the trajectory. The most illustrative differences whatsoever between both functionals can be read from Figures 2 and 3, where we see that J_{D1} (blue) reaches a higher lateral peak speed than J_{D2} just at the middle of the time period. The explanation for this is that by flipping between two opposite values of the acceleration (see Figures 2 and 3) until the final position is reached, the vehicle can get there with null lateral speed. However, for J_{D2} the speed will increase more smoothly from the first instant, but at the cost of not having null lateral speed at the end of the trajectory, which could mean a potential risk because of the inherent inertial dynamics.

Figures 4 and 5 show the trajectories traced by the two functionals for different values of the final time (t_f). Analyzing the results in this couple of Figures, we can conclude that functional J_{D1} can reach the optimum position at a later time but with a null lateral speed, whereas through functional J_{D2}

TABLE I
CONFIGURATION PARAMETERS FOR EVALUATION 1

`Evaluation parameter`	`Meaning`	`Value`
t_f	`Maximum time`	`2 s, 10 s, [1,5] s`
N	`Discretization factor`	`20 samples`
X_0	`Initial lateral position`	`1 m`
V_0	`Initial lateral speed`	`0 m/s`
a_0	`Initial lateral acceleration`	`0` m/s^2
W	`Road width`	`20 m`
n	`Type of functional`	`1(`J_{D1}`), 2(`J_{D2}`)`
$c(v_i)$	`Maximum absolute acceleration`	`3` m/s^2

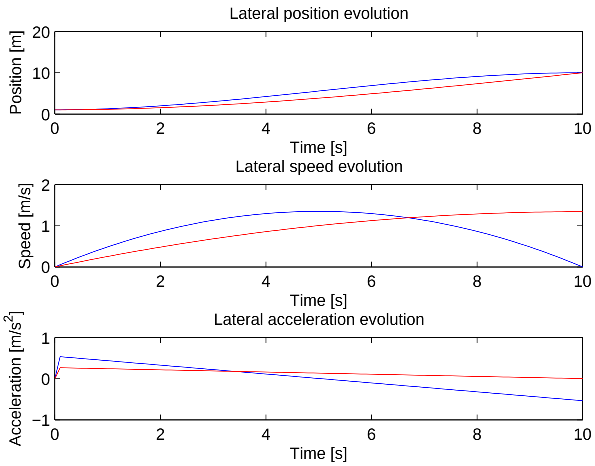

Fig. 2. Position, speed and acceleration J_{D1} (blue) and J_{D2} (red) ($t_f = 10$ s)

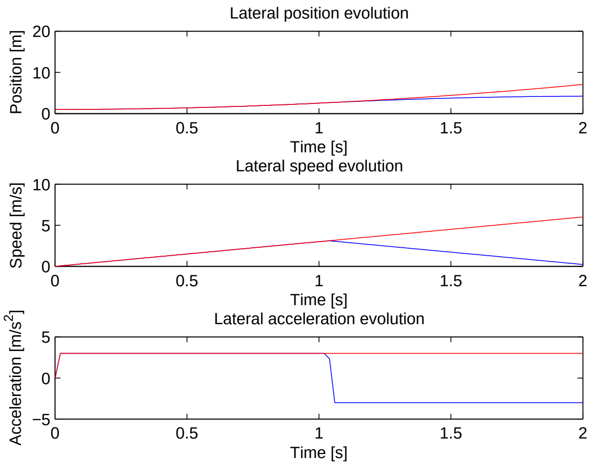

Fig. 3. Position, speed and acceleration J_{D1} (blue) and J_{D2} (red) ($t_f = 2$ s)

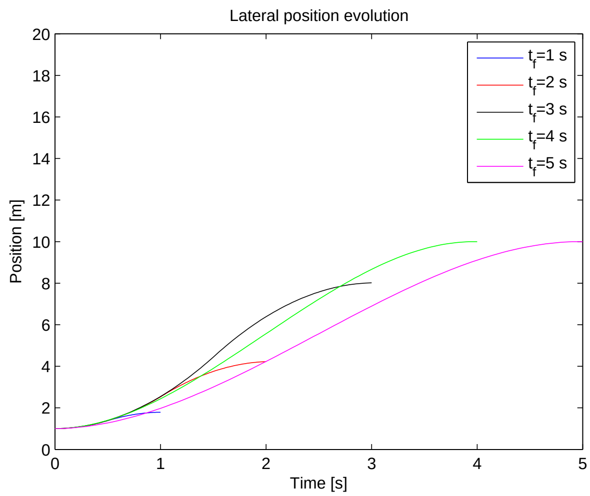

Fig. 4. Trajectory evolution for functional J_{D1} at different t_f

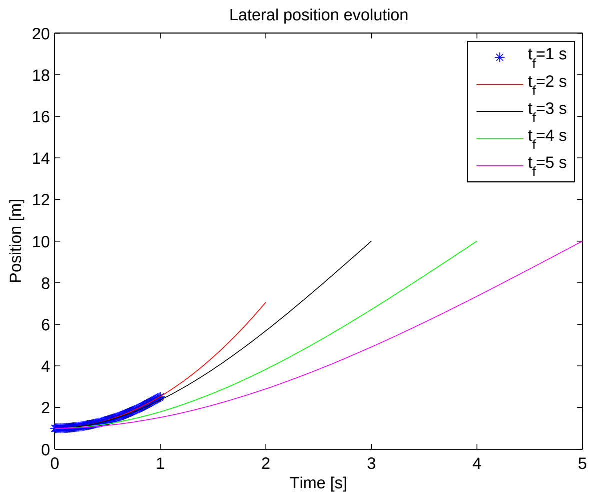

Fig. 5. Trajectory evolution for functional J_{D2} at different t_f

it is possible to reach the optimum lateral position earlier, but at the expense of having a non-zero speed. The first one will not have problems to follow the trajectory, but for the second one safety relies on how the inertial mobility will affect from the instant t_f onwards.

C. Instantaneous lateral distance maximization

In this part we focus on analyzing the properties of the trajectories when we use the instantaneous distance maximization as the optimization target (J_{D3} and J_{D4}). As we can

TABLE II

CONFIGURATION PARAMETERS FOR INSTANTANEOUS LATERAL DISTANCE MAXIMIZATION

Evaluation parameter	Meaning	Value
t_f	Maximum time	2 s, 10 s, [1,5] s
N	Discretization factor	20 samples
X_0	Initial lateral position	1 m
V_0	Initial lateral speed	0 m/s
a_0	Initial lateral acceleration	0 m/s^2
W	Road width	20 m
n	Type of functional	3(J_{D3}), 4(J_{D4})
$c(v_i)$	Maximum absolute acceleration	3 m/s^2

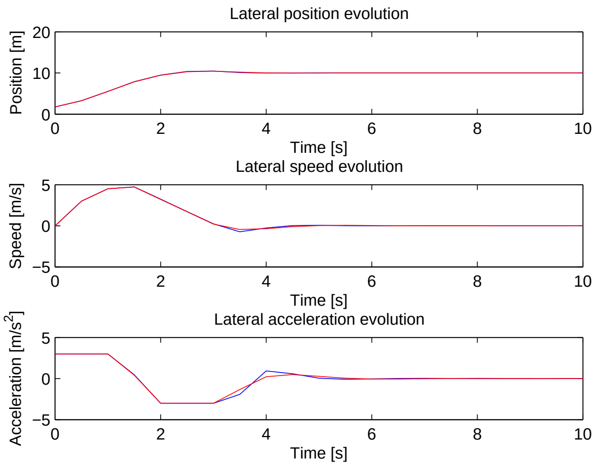

Fig. 6. Position, speed and acceleration J_{D3} (blue) and J_{D4} (red) ($t_f = 10$ s)

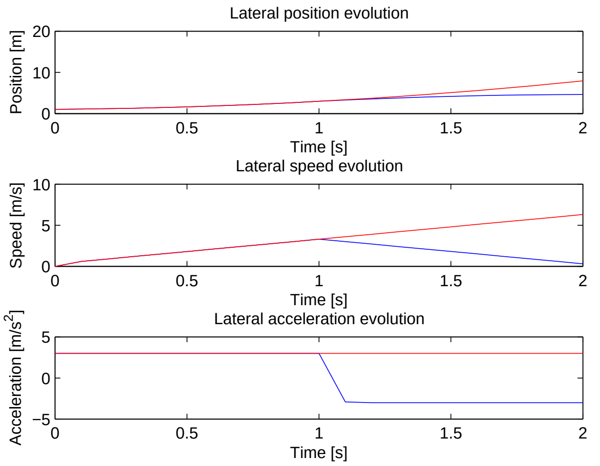

Fig. 7. Position, speed and acceleration J_{D3} (blue) and J_{D4} (red) ($t_f = 2$ s)

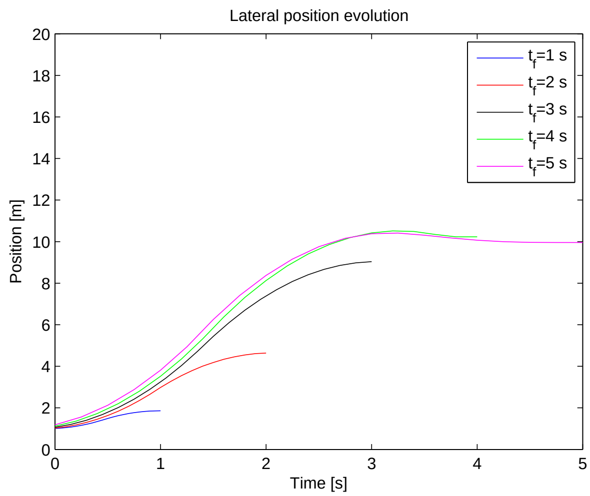

Fig. 8. Trajectory evolution for functional J_{D3} at different t_f

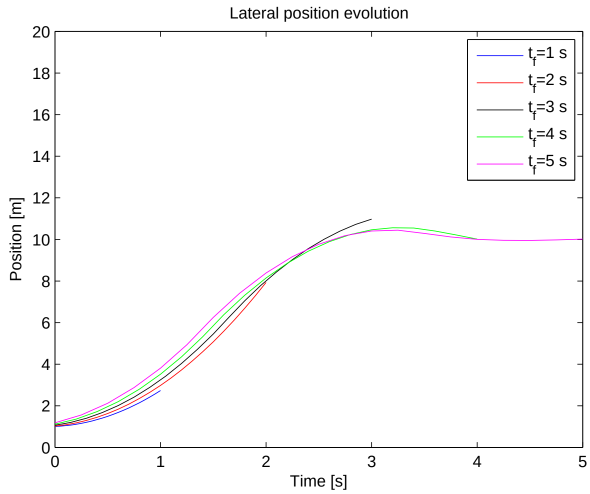

Fig. 9. Trajectory evolution for functional J_{D4} at different t_f

infer from Figures 6 and 7, in this case the main objective consists of adequating the lateral position as soon as possible in order to meet the requirements set by the functionals. This implies using alternatively the two highest (absolute) values of acceleration during the trajectory, which naturally makes the vehicle build a more sensitive path which can be obviously more affected by unstability. This is explained by assuming that the vehicle will try to adapt its lateral position instantaneously in order to maximize the lateral distance, thus it is possible that if there is a sudden change in the width of the road, or another car gets too close, the trajectory is modified very harshly, so it could undoubtedly end in a lateral collision. With J_{D1} and J_{D2} there could effectively be lateral collisions, but not due to unstability, because lateral distance is maximized regarding only the final lateral positions.

Regarding the results of the specific scenario of Table II for J_{D3} and J_{D4} we can see that for longer time intervals (Figure 6) until reaching the final position, a vehicle focus on achieving the final lateral position very quickly (thus the rapid changes in acceleration, which oscillate between the two extreme values although there is enough time to make a smooth maneuver). Besides, it is surprising that for both functionals the final lateral speed reached at the end of the path is null, and moreover, both functionals provide more or less the same behavior. On the other hand, if we analyze Figure 7 we see that for very short time intervals, the evolution of mobility is very similar to what we saw for the functionals J_{D1} and J_{D2}.

D. Discretization influence

Considering that using *Gradient Projection* to solve the optimization problems at hand implies the discretization of the trajectory in *N* stages (of constant acceleration), here we focus on the influence of the discretization factor *N*. The key test carried out here consists of comparing the trajectories traced by the four proposed functionals for a discretization step covering the interval 1 to 20 samples and evaluating the error (as the sum of the square of the difference between one curve and the reference one, set to *N*=100, see Eq. 8). We consider the curve *N*=100 as to be accurate enough for the calculation of the error evolution. Parameters' configuration for this case is shown in Table III.

$$\varepsilon\left(N\right)=\sum_{i=0}^{N_{ref}}\left(x_N\left(i\frac{t_f}{N_{ref}}\right)-x_{N_{REF}}\left(i\frac{t_f}{N_{ref}}\right)\right)^2 \qquad (8)$$

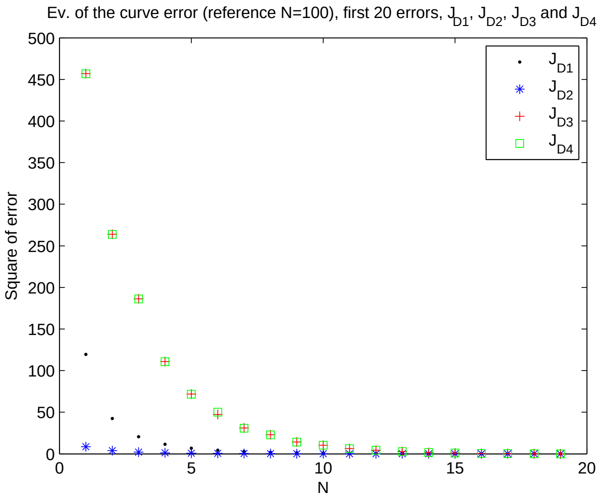

Fig. 10. Influence of discretization factor N

where $\varepsilon\left(N\right)$ refers to the square error for an *N*-discretized curve $(x_N(t))$, in comparison with the N_{ref}=100 reference trajectory $(x_{N_{ref}}(t))$.

Having a look at the Fig. 10 it is easy to notice the correct results obtained by the algorithm, in which the highest errors are located for low values of the discretization factor. Observing this graph it is also straightforward to infer that setting a value of *N*=20 should be enough to reach a proper resolution in the curve for the four functionals. We can see that the functional J_{D1} needs a higher value for the discretization factor than that of J_{D2} to achieve the same level of accuracy. This can be explained taking into account that to reach null speed at the end of the trajectory, functional J_{D1} needs to take higher values of the instantaneous acceleration, while functional J_{D2} does not need so high values because its trajectory evolution is smoother, at the expense of not having null speed when reaching the final position. On the other hand, both functionals regarding instantaneous lateral distance maximization (J_{D3} and J_{D4}) show that they need a higher value of the discretization factor to reach the same level of accuracy. This is due to the higher values of acceleration which a vehicle needs when it performs the optimization of the trajectory according to the instantaneous lateral distance.

E. Connection to CCA

Analyzing the procedure of automatic trajectory tracing for one vehicle is the first step to completely evaluate a fully interconnected traffic scenario where vehicles interchange information to adapt their trajectories to their own paths and the unpredictable phenomena which could alter the normal transit. Just one vehicle does not imply a real cooperative system to avoid accidents, but this case gives us a valuable insight into the establishment of optimum trajectories which can essentially improve traffic conditions (in terms of safety and traffic efficiency). When referring to CCA, we can infer from the last subsection that the necessary discretization of trajectories will require a high frequency of beacons' interchange between vehicles in order to keep an updated history of the mobility evolution of cars along the path. IEEE 802.11p [3] is based on a CSMA (*Carrier Sense Multiple Access*) whose performance degrades as the number of transmitting nodes increases [9]. Thus, it is very important to determine which are the minimum requirements which this system should have in order to allow an excellent performance (necessary in situations in which having this kind of information is critical). This is something we will treat in a future work.

IV. CONCLUSIONS

From the present work we can conclude that the task of reorienting the vehicle when approaching a certain objective position on the road can be challenging. Here we have presented some results regarding a mathematical evaluation of how to trace the trajectory of a vehicle which wants to relocate its lateral position before reaching a certain position on the road determined by t_f. The four functionals present both advantages and drawbacks. Functionals J_{D1} and J_{D2} provide

TABLE III

CONFIGURATION PARAMETERS FOR THE CASE OF DISCRETIZATION INFLUENCE

Evaluation parameter	Meaning	Value
t_f	Maximum time	5 s
N	Discretization factor	[1,20] samples
X_0	Initial lateral position	1 m
V_0	Initial lateral speed	0 m/s
a_0	Initial lateral acceleration	0 m/s^2
W	Road width	20 m
n	Type of functional	1(J_{D1}),2(J_{D2}), 3(J_{D3}),4(J_{D4})
$c(v_i)$	Maximum absolute acceleration	3 m/s^2

a better stability during the trajectory because they only update the mobility parameters according to the final lateral position. J_{D3} and J_{D4}, however, base their optimization procedure on updating the position in terms of the instantaneous distance, which requires faster changes in the mobility evolution. On the other hand, whereas J_{D1} is really useful for higher values of t_f (since apart from reaching the optimum position it arrives with null speed), J_{D2} reaches the last position with a speed higher than zero (and sometimes too high). However, for low distances until the optimum position, J_{D2} can reach this position earlier, while the other functional focuses its attention on reaching the last position at zero speed, not caring as much about how far we are from the optimum position we want to reach. From these results we can deduce that in general optimizing in terms of the final lateral distance can be better to avoid very rapid changes in acceleration which could imply the need to have higher values of the discretization factor *N*. However, we have not explored the whole amount of scenarios which appear from this premise, and in certain cases using functionals J_{D3} and J_{D4} (or a derivation of them) could be more convenient. In a future work we will investigate a vaster amount of scenarios to completely characterize the performance of the four functionals (and possible derivatives).

Last but not least, the problem solved here corresponds to a scenario with no obstacles or other vehicles. It's our aim to study the problem for a higher number of vehicles (more differential equations) and more obstacles. This will cause us to analyze functionals whose gradients are semi-convex. For this reason, depending on the inital configuration values, the algorithm may not converge to the global minimum. Expressing the functionals slightly different or even using other tools for solving this kind of optimum control problems will be probably necessary if we want to provide the best solution for each specific scenario we might have to cover.

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REFERENCES

[1] "Annual report of accidents". *National spanish traffic administration*, (DGT, Direccion General de Trafico). Ministry of inner affairs (Ministerio del Interior). Government of Spain (Gobierno de Espana) 2010. http://dgt.es/portal/en/seguridad_vial/estadistica/publicaciones/anuario_estadistico/

[2] Ili, S., Albers, A. and Miller, S. (2010), Open innovation in the automotive industry. R&D Management, 40: 246255.

[3] Family of protocols for WAVE architectures 1609.1, 1609.2, 1609.3, 1609.4. IEEE 802.11p, Wireless Access for Vehicular Environments Draft Standard. IEEE 2010.

[4] Venkatraman, A.; Bhat, S.P.; , "Optimal Planar Turns Under Acceleration Constraints". Decision and Control, 2006 45th IEEE Conference on, pp.235-240, 13-15 Dec. 2006.

[5] Anisi, D. A.; Hamberg, J.; & Hu, X. "Nearly time optimal paths for a ground vehicle". Journal of Control Theory and Applications, 1(1), 28. 2003.

[6] Tjahjana, H.; Pranoto, I.; Muhammad, H.; Naiborhu, J.; Miswanto. "The Numerical Control Design for a Pair of Dubin's Vehicles". Computer Science - Robotics, I.2.8. April 2008.

[7] Dingle, P.; Guzzella, L.; , "Optimal emergency maneuvers on highways for passenger vehicles with two- and four-wheel active steering". American Control Conference (ACC), 2010 , pp.5374-5381, June 30 2010-July 2 2010.

[8] Kirk, D.E. Optimal control theory: An introduction. 1971. Prentice-Hall, London. 453 p.

[9] Eichler, S.; "Performance Evaluation of the IEEE 802.11p WAVE Communication Standard". Vehicular Technology Conference, 2007. VTC-2007 Fall. 2007 IEEE 66th , pp.2199-2203, Sept. 30 2007-Oct. 3 2007.