

# A Stochastic Model for Design and Evaluation of Chain Collision Avoidance Applications

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## Abstract

Cooperative/Chain Collision Avoidance (CCA) is an important type of safety-related applications of vehicular ad hoc networks (VANETs). They provide collaborative communication capabilities to vehicles in order to reduce the number of accidents on the road. Simulation is the usual choice to evaluate these systems. However, current simulation tools based on car-following models cannot be seamlessly used to simulate accidents, as we discuss here. Therefore, in this paper we propose the use of a stochastic model as an alternative to simulation for the design and performance evaluation of such applications. The model enables the computation of the average number of collisions that occurs in a platoon of vehicles, the probabilities of the different ways in which the collisions may take place, as well as other statistics of interest. The suitability of the model for evaluating CCA applications is shown by comparing our results with other authors' simulation results. Additionally, it can be used at an early stage to shed relevant guidelines for the design of CCA applications, by disclosing the influence of kinematic parameters on the collision process. To exemplify it, we provide an evaluation of different types of CCA applications in two scenarios, a freeway and an urban scenario.

*Keywords:* Vehicle safety, vehicular communications, chain collision, vehicle platoon, collision avoidance, stochastic model, road accidents

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## 1. Introduction

Inter-vehicle communications based on wireless technologies pave the way for innovative applications in traffic safety, driver-assistance, traffic control and other advanced services which will make future Intelligent Transportation Systems (ITS) possible. Improvement of traffic safety by cooperative vehicular applications is one of the most promising benefits of Vehicular Ad Hoc Networks

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<sup>\*</sup>This research has been supported by the MICINN/FEDER project grant TEC2010-21405-C02-02/TCM (CALM) and Fundación Séneca RM grant 00002/CS/08 FORMA. It is also developed in the framework of "Programa de Ayudas a Grupos de Excelencia de la Región de Murcia, de la Fundación Séneca, Agencia de Ciencia y Tecnología de la RM". C. Garcia-Costa acknowledges the Fundación Séneca for a FPI (REF 12347/FPI/09) pre-doctoral fellowship.

(VANETs). Cooperative/Chain Collision Avoidance (CCA) applications [1, 2] are a particular implementation of safety related services aimed at avoiding, or at least mitigating, chain collisions in a convoy of vehicles. In order to develop properly such applications, the influence of the different driving parameters on the event of a vehicle collision must be assessed at an early design stage.

Simulation is an essential tool to design and evaluate these kind of applications. However, there is a number of issues related to this approach. First, it usually requires the integration of networking and traffic simulation tools, which is not mature yet and requires further work [3]. But, more importantly, available simulation tools are not directly suitable to model accidents and so cannot be effortlessly used for the design of cooperative applications. The reason is that current traffic simulation tools are designed for normal traffic conditions and are based on mobility models that are specifically developed to avoid vehicle crashes, for example, a common metric for the quality of a car-following model is that it is intrinsically *collision-free*. Therefore, those models have to be modified to account for collisions which is either not a straightforward task and may lead to unexpected results or it is difficult to set up controlled experiments. Some of these limitations are pointed out and discussed in the following sections.

In a previous work [4] we derived a stochastic model for the number of accidents in a platoon of vehicles equipped with a CCA system. The model enables the computation of the average number of collisions that occur in the platoon, the probabilities of the different ways in which the collisions may take place, as well as other statistics of interest. In this paper, we discuss its potential as a numerical evaluation tool and as an alternative to simulation, specially at early stages of development. Our goal is to illustrate its use by providing and thoroughly discussing application examples. First, the stochastic model is described as well as the different metrics it can provide. Its limitations are also discussed. Next, we show how it can be used as a performance evaluation tool and check the validity of the results it provides by comparing them with available independent results. Finally, the model can be used at an early stage to shed relevant guidelines for the design of CCA applications, by disclosing the influence of kinematic parameters on the collision process. To exemplify it, we provide an evaluation of different types of CCA applications in two scenarios, a freeway and an urban scenario. Those scenarios have been carefully characterized by extracting appropriate parameters' distributions from open literature. The results suggest that enabling a coordinated braking policy that removes the variability in deceleration and driver reaction time should be the main concern of a CCA application. It should be noted that particular numerical results have to be considered as upper bounds on the expected number of accidents, since the model is based on the strong assumption that vehicles cannot change lane to avoid the crash (worst case). However, even for the generic scenarios and simplified systems used as examples, it is able to provide a reasonable qualitative insight of the relative benefits of different CCA approaches.

The remainder of this paper is organized as follows. In Section 2 relevant related work is reviewed. Next, in Section 3, the limitations of current simulators

are discussed before providing an overview of the stochastic model. Finally, some of the performance metrics that the model can provide as output are described, as well as its current limitations. In Section 4 the suitability of the model for evaluating the performance of CCA applications is shown by comparing it with previous results. Section 5 provides, as an illustrative example, the evaluation of different CCA systems under two scenarios. Finally, conclusions and future work are remarked in Section 6.

## 2. Related Work

The concept of Automated Highway Systems (AHS) goes back several decades and its safety benefits have been studied in the past years [5]. The motion of a platoon of vehicles is usually described as an interconnected (automated or not) system, where one or more leading vehicles influence the driving behavior of the follower. Platoon safety comes as a result of proper stability of the platoon in the presence of perturbations, called string stability. The basis of string stability and safety performance guarantees can be found in [6].

In the absence of safety guarantees collisions may occur and they have been studied mainly by modeling its frequency [7], severity [8] or physical process [9, 10, 11]. In the latter case, very detailed models of vehicle motion and collision dynamics can be found [10, 11], but the equations are completely deterministic, whereas in reality, randomness is always present as an effect of human behavior or noisy operation introduced by sensors or other reasons. To account for it, the usual methodology is to evaluate deterministic models by applying a Monte Carlo or stochastic analysis over an extensive range of their parameters [8, 10, 12].

On the contrary, in this paper we use the stochastic model we proposed in [4], which assumes that kinematic variables are random. Indeed, it has been extended for this paper in order to support all variables being random simultaneously and arbitrary distributions for the inter-vehicle spacing. A similar stochastic approach can be found in [9, 13], where the authors assume the “effective” braking is a random variable and analytically compute the probability and expected number of primary collisions and the relative speed at impact in a platoon of vehicles. The inter-vehicle distance and speed of all vehicles are identical, unlike our model, where we consider both parameters as random variables with a known probability distribution function. Their analysis is also based on some strong assumptions, e.g., the expected number of total collisions in the string is assumed to be proportional to the expected number of primary collisions. Therefore, they provide a lower bound on the expected number of collisions. Secondly, they assume that a collision will definitely occur if the deceleration of a following vehicle is less than that of its immediate predecessor. On the contrary, our model considers all the ways in which a collision may occur and provides the average probability of each type of collision, as well as an upper bound for the total number of collisions. In Section 4 we validate our model by comparing with theirs.

Carbaugh et al. [5] have modeled rear-end crashes and related them to the capacity of AHS. Our work follows a similar approach in several aspects, though it has relevant differences as well. Unlike our work, they restrict themselves to only primary collisions involving only two vehicles. They consider random variables for speed, braking capabilities and reaction times, as we do. However, we introduce them in the analytical model and compute the performance metrics, whereas they discretize the distributions and evaluate all the parameter combinations with Monte Carlo simulations. In both works, parameters have been carefully extracted from open literature. Finally, in both cases, different types of cooperative vehicle systems have been evaluated. The main differences come from the modeling of the cooperative systems: in [5] they are essentially distinguished by the reaction time whereas the speed and inter-vehicle spacing are fixed and constant. In our case, different cooperative systems are not only assigned different reaction times, but also braking and speed behavior, which do not need to be constant. Another major difference is that they explicitly relate safety metrics and road capacity, whereas we use capacity as an independent parameter and do not explicitly mention it. That is, capacity is implicitly given by the random variables used to model the states variables (inter-vehicle spacing and speed) in the scenarios used in Section 5 and Table 2. For instance, the free-flow highway traffic corresponds approximately to a capacity of 2347 vphpl, by substituting the average speed and inter-vehicle spacing in eq. (2) in [5].

With the recent assignment of bandwidth and standardization of communications for vehicular networks, research interest on cooperative vehicular applications has grown again. Kato *et al.* [14] show the feasibility and potential of the technologies for the cooperative driving. In fact, improvement of traffic safety by cooperative vehicular applications is one of the most promising benefits of vehicular ad hoc networks. In a recent work [15] the authors propose an inter-vehicle communication framework for the cooperative active safety system whose operation is based on the dissemination of each vehicle's state information through a wireless network.

As a particular case of cooperative driving, Cooperative Collision Avoidance (CCA) techniques have received special attention in recent years. With CCA systems a fast dissemination of warning messages to the vehicles in the platoon enables them to promptly react in emergency situations. In this way the number of car accidents and the associated damage can be significantly reduced. In [16] the authors identify the application requirements for vehicular cooperative collision warning and achieve congestion control for emergency warning messages based on the application requirements. The authors in [17] develop three cooperative collision warning safety applications: a forward collision warning assistant, an intersection assistant and a blind-spot and lane change "situational awareness" assistant. In [18] we can find a performance evaluation study of cooperative collision warning applications using the DSRC wireless standard. All these studies focus on the communication or implementation aspects of the application. They use simulations that do not involve crashes and do not provide any results related to safety and so the problems with accurate simulation

of crashes that we discuss in the next section do not arise. Biswas *et al.* [19] present an overview of highway cooperative collision avoidance and its implementation requirements in the context of a vehicle-to-vehicle wireless network, primarily at the Medium Access Control (MAC) and the routing layers. More recently, Taleb *et al.* [20] proposed an effective collision avoidance strategy for vehicular networks which forms clusters of vehicles that belong to the same group. They also design a risk-aware medium access control (MAC) protocol to increase the responsiveness of the proposed CCA scheme. These studies do evaluate safety aspects of the systems, but as a complement to the communication evaluation. So they develop simple ad hoc mobility models instead of traffic simulators or car-following models, and again the problems of accurate simulation of accidents do not show clearly. Several car-following mobility models have been proposed and analyzed [21, 22], also in the context of VANET simulation [3]. A thorough analysis of car-following models for accident simulation can be found [23], where the authors propose a car-following model that includes by design accidents behavior as well. Its integration with current simulators still require to solve additional issues and at the moment has not been incorporated to available tools.

### 3. Stochastic modeling of accidents and simulation

Evaluating CCA applications for vehicular networks requires, as a previous step, appropriate modeling of accidents and driver behavior in such situations. That is, CCA designers need to understand the processes that lead to crashes and the influence of different system variables under such circumstances in order to cooperatively take preventive measures. However, as we discuss in the next section, available simulation tools are not directly suitable to reproduce such processes and so cannot be effortlessly used for designing cooperative CCA applications.

Whereas modeling of vehicle structural deformation and occupant injuries has been widely studied under different contexts [24] and modeling tools are available, current traffic simulation tools are focused on normal traffic conditions and are based on mobility models that are specifically developed to avoid vehicle crashes, for example, a common metric for the quality of a car-following model is that it is essentially *collision-free*. Therefore, those models have to be modified to account for collisions which is either not a straightforward task and may lead to unexpected results or it is difficult to set up controlled experiments. In Section 3.1 we further discuss some concerns which arise when using popular simulators and mobility models to simulate crashes. Later, in Section 3.2 we describe our stochastic model and discuss its limitations in Section 3.3.

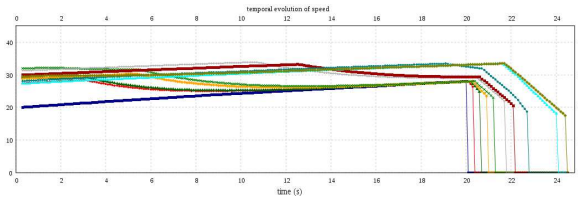
#### 3.1. Simulation of accidents with current tools

Simulators based on macroscopic magnitudes are not appropriate to simulate accidents, so it is necessary to resort to micro-simulation. Most popular micro-simulation tools [25, 26] are based on car-following models [21, 23]. In particular,

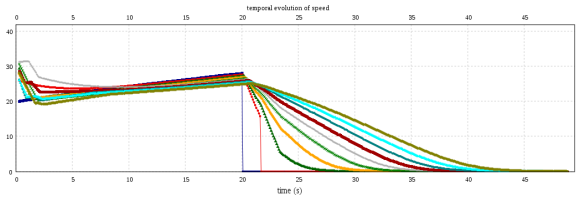
the Gipps model is used by AIMSUM [25] and a modified Krauss model is used by SUMO [26]. Both models are called *safety distance models* because it is assumed that drivers try to keep a safety distance with the preceding vehicle to avoid accidents. Both of them use a reaction time  $\tau$  parameter and an estimation of the preceding car comfortable deceleration in order to compute the next step speed, which is instantaneously updated, together with a maximum comfortable acceleration and deceleration. The Gipps model is collision free as long as the comfortable deceleration of the preceding vehicle is not underestimated [23]. The Krauss model adds a stochastic perturbation to the acceleration but it is collision free because the speed is bounded by a *safety speed* at each updating speed. Moreover, in its implementation in SUMO, every vehicle (or driver) *knows exactly* the deceleration rate and reaction time of the preceding vehicle. Finally, the Intelligent Driver Model (IDM), and its imperfect driver variant, the Human Driver Model (HDM) [22] are becoming popular for simulation in the last years. The former one is collision free by design unless a maximum deceleration is used.

Therefore, in order to simulate accidents, and more particularly chain collisions, one has to modify the models. For the Gipps model, a first obvious approach is to limit the maximum deceleration and remove the safety speed constraint from the model. To obtain a more realistic behavior, the reaction time of each driver can be randomized as well as the estimation of the preceding vehicle deceleration. These changes allow to simulate accidents to some extent, but setting up controlled accident experiments is still hard. The main reason behind is that car-following models lead to an equilibrium state where either all the platoon accelerations are zero or strong instabilities with oscillations occur [21]. Then, if a simulation is started with initial conditions different from those of the equilibrium state it results in an initial transient where all the cars immediately adapt their speed to that prescribed by the model, which tend to avoid collisions. In other words, the platoon behaves as if there is a cooperative safety application in place which automatically and instantaneously dictates the needed speed. For example, if one is interested in the influence of small inter-vehicle gap on the accidents, the model parameters and initial conditions have to be carefully adjusted to overcome the automatic reconfiguration of the platoon. And in many cases it is likely that the model itself has to be tuned as we discuss next. The opposite situation is also common, that is, initial transient leads to strong instabilities, which propagate backwards and result in unexpected crashes even before the programmed emergency event. The IDM/HDM model is particularly sensitive to initial deviations from equilibrium state.

In addition, the influence of model and simulation parameters on the results is not always clear and in some cases their interpretation is different. For instance, Gipps and Krauss models describe the parameter  $\tau$  as a reaction time. HDM also introduces a reaction time parameter. One would expect high reaction times to increase the risk of accidents. That is, let us consider a simple scenario with two cars where the leading one suddenly decelerates at a high rate. Reaction time is commonly assumed to be the time elapsed since the leading vehicle starts to brake and the following one perceives the change and starts to



(a) Krauss model with  $\tau = 0.2$  s.



(b) Krauss model with  $\tau = 1.2$  s.

Figure 1: Temporal evolution of vehicle speeds with SUMO and a Krauss model with different reaction time  $\tau$ . If there is a collision, speeds drop instantaneously to zero.

brake itself, that is, higher reaction times would lead to late brake and more dangerous situations. This is actually the interpretation and behavior of the HDM model. However, in the Krauss model the leader reaction time<sup>1</sup> determines what is considered the safety speed by the follower: a higher reaction time makes the follower to choose a lower safety speed, which makes the model collision free in normal situations. A first objection is that knowledge of the leader reaction time does not seem realistic. But more importantly, that difference makes the results quite the opposite of expected, since in practice it determines the *aggressiveness* of the driver style. This is a consequence of the way the model is constructed. In equilibrium cars follow each other with a time headway that is equal to the reaction time and the safety speed is computed assuming normal conditions comfortable deceleration. Thus, low reaction times lead to short time headways. When an accident occurs, decelerations much higher than the comfortable one can be expected. In that case, short time headways result in more accidents. Fig. 1 exemplifies this behavior in an extreme scenario where a platoon of 9 vehicles follow a leader which at  $t = 20$  stops instantaneously. As can be seen in Fig. 1(b), drivers are more conservative. At the beginning all the vehicles reduce their speed to comply with the safety speed and keep higher time headways, which in the end result in few collisions. It also exemplifies the initial automatic adjustment discussed in the previous paragraph. On the contrary, in Fig. 1(a) it is shown how vehicles at the beginning accelerate to reduce its headway and start to decelerate later, leading to multiple collisions. In this sense, IDM/HDM provides more flexibility since it separates desired gap

<sup>1</sup>Actually, reaction time is the same for all vehicles in the Krauss model.

and reaction time in a more realistic way.

In any case, a thorough analysis of car-following models in the context of accidents is out of the scope of this paper, interested readers can see [23]. Our goal here is to simply point out some of the subtleties involved in accident simulation with current simulation tools. To the best of our knowledge, the only available car-following model that includes by design accident behavior is described in [23], but integration with current simulators still require to solve additional issues and at the moment have not been incorporated to available tools. In summary, simulation of accidents with current simulation tools is neither straightforward nor obvious and need careful design and adjustments, in addition to the usual drawbacks of simulation.

### *3.2. Stochastic collision model for vehicles equipped with communications*

In the previous section some of the difficulties of accident simulation with current tools have been discussed. In this section we propose, as an alternative to simulation, to numerically evaluate the influence of different system parameters as reaction time, speed or deceleration capabilities on the number of accidents and other metrics. The numerical evaluation is based on a stochastic model for chain collisions that has been derived in a previous paper [4]. The model assumes that all vehicles are equipped with a CCA that sends a warning message when an accident occurs. In that case, it can be assumed that drivers react as soon as they receive the warning message, and they start braking just after the time they need to be aware of the danger (reaction time). Hence, the total delay is the sum of the warning message reception and reaction delays. This reaction is independent of the preceding vehicle behavior. The main practical utility of this model lays on its ability to quickly evaluate numerically the influence of different parameters on the collision process without the need to resort to complex simulations at a first stage. Such an evaluation provides relevant guidelines for the design of vehicular communication systems as well as chain collision avoidance (CCA) applications. The limitations of the model as well as the performance metrics it can provide are discussed in next section.

A complete description of the developed stochastic model can be found in [4], but for completeness we outline it here as follows. We consider a platoon (or chain) of  $N + 1$  vehicles driving in convoy (see Fig. 2), where each vehicle  $C_i$ ,  $i = 0, \dots, N$ , moves at constant speed  $V_i$ . The leading vehicle,  $C_0$ , faces an emergency situation and immediately brakes at a high deceleration or emergency deceleration and sends a warning message to the following vehicles. The remaining vehicles start to brake at deceleration  $a_i$ , which depends on the CCA system or policy under evaluation, when they are aware of the risk of collision, that is, after a time lapse  $\delta_i$ . Let us note that  $\delta_i = T_{r,i} + T_{m,i}$  includes both a reaction time and a message reception time, and so it allows to evaluate both contributions separately. Assuming that every vehicle has the same average length  $L$ , and its position is given by the  $x$  coordinate of its front bumper, the initial inter-vehicle spacing is  $s_i = x_i - (x_{i-1} + L)$ . To test the worst case situation, vehicles cannot change lane or perform evasive maneuvers. This is a



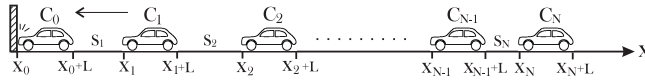


Figure 2: The scenario under consideration.  $x_i$  is the position of vehicle  $C_i$ ,  $L$  is the average length of vehicles and  $s_i$  represents the inter-vehicle spacing.

worst-case assumption, commonly used in literature [5, 9], that leads to upper bounds in the results.

The model needs five inputs: the number of vehicles, the distribution of the inter-vehicle spacing and vectors for the speeds, delays and decelerations. The first three ones define the initial state whereas the last two are usually controllable. That is, we can imagine that at the time instant of the emergency event we take a snapshot of the system. From this snapshot we extract the speed of each vehicle and the distance between two consecutive vehicles. Therefore, the initial state of the system is defined by the speeds  $\{V_i\}_{i=0,\dots,N}$  and inter-vehicle spaces  $\{s_i\}_{i=1,\dots,N}$ , which will be called state variables. On the other hand, the delays before braking  $\{\delta_i\}_{i=0,\dots,N}$  and the deceleration rates  $\{a_i\}_{i=0,\dots,N}$  will depend on the decisions made by the drivers after the time instant of the emergency event, which may be influenced by a CCA application and will be called control variables. We assume that at least inter-vehicle distance is a random variable, but the remaining variables can be considered random or assigned constant values as it is discussed in Sect. 3.3. The output of the model is one or more of the performance metrics discussed in the next subsection, as the average number of accidents.

The first step consists of the computation of the vehicle collision probabilities. This is the main contribution of our model and is calculated recursively. For each vehicle  $C_i$ , starting from the leading one, we compute its collision probability  $p_i$ , which is based on the average distance traveled by the preceding vehicle  $\bar{l}_{i-1}$ :

$$p_i = P(d_{s,i} \geq \bar{l}_{i-1} + s_i) = F(d_{s,i} - \bar{l}_{i-1}), \quad i = 1, \dots, N, \quad (1)$$

where  $F$  is the cumulative distribution function (cdf) of the inter-vehicle spacing and  $d_{s,i}$  is the distance needed by vehicle  $C_i$  to completely stop, which is defined by:

$$d_{s,i} = \frac{V_i^2}{2a_i} + V_i\delta_i. \quad (2)$$

In a second stage we compute the average distance traveled by the current vehicle, which is then used in the computation of the next vehicle collision probability. This computation takes into account the different ways a collision between two vehicles traveling in the same direction may occur: (1) vehicles have not started to brake; (2) only one of them is braking; (3) both of them are braking; or (4) the front vehicle has stopped. We also assume that when two vehicles collide, they instantly stop at the point of collision [8]. We compute the average distance traveled in each type of collision,  $d_{c_j,i}$ , and its probability

of occurrence,  $q_{c_j,i}$ , as follows:

$$d_{c_j,i} = \frac{1}{q_{c_j,i}} \int_{inf_j}^{sup_j} D_{c_j,i}(x) f(x) dx, \quad (3)$$

$$q_{c_j,i} = P(inf_j \leq s_i \leq sup_j) = F(sup_j) - F(inf_j), \quad (4)$$

for  $i = 1, \dots, N$ ,  $j = 1, \dots, 4$ , where  $f$  represents the probability distribution function (pdf) of the inter-vehicle spacing distribution and  $D_{c_j,i}(x)$  the distance traveled by  $C_i$  when the inter-vehicle spacing is  $x$  and it collides in the way  $j$ , which is calculated in [4], as well as the limits of integration  $inf_j$  and  $sup_j$ .

Then, we compute the total average distance traveled by the current vehicle using the weighted sum:

$$\bar{l}_i = d_{s,i}(1 - p_i) + d_{c_1,i}q_{c_1,i} + d_{c_2,i}q_{c_2,i} + d_{c_3,i}q_{c_3,i} + d_{c_4,i}q_{c_4,i}, \quad (5)$$

for  $i = 1, \dots, N$ .

Let us remark that in our model the inter-vehicle spacing is always assumed to be a random variable *independent* of the other variables. This is a reasonable assumption for delays and decelerations, but not for initial speeds. In fact, this is a limitation of the model which is further discussed in Sect. 3.3. On the other hand, independence is also an advantage since it allows to effortlessly evaluate different distributions for inter-vehicle spacing just by substituting the appropriate probability density function in the equations above.

The system may result in  $N+1$  possible outcomes ( $N$  collided vehicles and 0 successfully stopped vehicles,  $N-1$  collided and 1 successfully stopped vehicle, and so on). So finally, we compute the probabilities of going from the initial state (the time instant when the emergency event occurs) to each final outcome and we compute the average number of collisions in the chain by using the weighted sum of these probabilities.

As a final note, in our previous paper [4] we only considered exponential inter-vehicle spacing and two random parameters (one deterministic at a time). For this paper we have extended our model and now it can evaluate cases with all parameters being random and arbitrary distributions for inter-vehicle spacing. We do not explicitly show here the modifications due to lack of space.

### 3.3. Limitations and performance metrics

Our model was derived under the assumption that at least the inter-vehicle spacing is a random variable. The other parameters (velocity, delay and deceleration) can be assigned deterministic or random values before executing the numerical evaluation. However, introducing too much randomness causes unrealistic results. In fact, it results in a pessimistic estimation of the metrics.

Inter-vehicle distances and velocities represent the state of the system when the incident occurs, and so they should be considered random variables in most cases, though determining their distributions and ranges require a proper characterization of the scenario of interest. Accelerations and delays can be controlled

by different means after the incident, and so depending on the application evaluated they can be considered constant or assigned particular values. Indeed, different CCA systems are mainly characterized by how they are modeled. As an example, let us consider two different CCA systems. The first one is simply characterized by a warning message delivered to the platoon that makes drivers start braking. In this case, both delay and deceleration should be considered random variables modeling driver reaction time and human-operated braking respectively. The second CCA system is a fully automated braking system that takes over the driver operation and applies a constant deceleration. In that case, both delays and decelerations could be considered deterministic.

Randomness allows the occurrence of situations which rarely occur in reality for both state and control variables. For the state variables, independence may result in samples where a vehicle is traveling very near to its front vehicle and much faster than it, which is unlikely to happen in reality<sup>2</sup>. Actually, there is a correlation between the distance from a vehicle to the preceding one and the relative velocity between them [5]. Unfortunately, this correlation cannot be represented in the model in its current form, so the number of accidents computed by it is an overestimation of the actual number of accidents that may occur in a real situation. For the control variables, the model does not take into consideration that the driver usually reacts to variations in the driving conditions of the preceding vehicle after the incident, as in a car-following approach. That is, if we assign pure random decelerations it may result in a driver which decelerates much softer than its preceding vehicles, leading to a crash. In a real situation, the follower would apply a stronger deceleration to avoid the crash. Fortunately, these cases can be corrected since the control variables can be freely assigned, as we describe in Sect. 5. Finally, as we said previously, the assumption that vehicles cannot change lane or perform evasive maneuvers, often found in literature [5, 9], results again in an overestimation. Anyway, in the following sections we show that in spite of those limitations the model is suitable for evaluating the performance of CCA-based applications in different scenarios and the influence of the main kinematic parameters on the number of vehicle collisions.

A variety of performance metrics can be provided by the model, either direct metrics, typically used in literature [19, 20], as well as more specialized ones, indirectly derived from the former ones, as follows:

- **Percentage of accidents.** This is a direct global metric that computes the average percentage of collided vehicles in the chain.
- **Relative distance.** It provides for each vehicle in the chain the average distance to the preceding car after stop (in case of collision the relative distance is 0). This metric may be considered as a measure about the margin of safety available to the vehicles.

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<sup>2</sup>Actually its likeliness is arguable since this case does occur when a vehicle is preparing to overtake its front one.

- **Relative speed.** It provides for each vehicle in the chain the average relative speed with respect to the preceding vehicle at the time of collision (in the absence of collision the relative speed is 0). This metric may be considered as a measure about the severity of collisions.
- **Types of collisions.** As previously mentioned, collisions can occur into four different ways in a single-lane situation. The average probability of each type of collision can be provided, which can be used by itself or as a weight factor for other derived metrics.
- **Accident severity functions.** These are specialized metrics derived by weighting accident severity indexes with probabilities of types of collisions and other metrics. As an example, a collision between two vehicles in movement can be assigned a higher severity than a rear collision with a stopped one. The average severity index results from weighting them by the probability of either type of collision. In addition, the average relative speed at the collision can be used to weight again the severity index.

A number of other metrics can be constructed, depending on the application under evaluation, though in next sections we only use the first three ones.

#### 4. Evaluation of CCA applications

As said previously, the evaluation of this kind of applications is usually conducted by time-consuming simulations and needs a considerable prior development effort. Using our model, one can quickly evaluate numerically the performance of a CCA application under different situations. In this section, we show that our model can provide similar results, and so it is a good alternative to simulation, by comparing its output with the results reported by previous performance evaluation of CCA mechanisms. The first one is a basic CCA system proposed by Biswas *et al.* [19] and the second one is a more sophisticated mechanism, the Cluster-based Risk-Aware CCA (C-RACCA) scheme proposed by Taleb *et al.* [20], which are briefly described next.

- **CCA:** Upon occurrence of the emergency event, the leading vehicle rapidly decelerates (emergency deceleration, see Table 1) and starts sending emergency warning messages to all vehicles behind it. These messages are forwarded in a multihop manner in order to ensure a complete coverage within the platoon. Upon reception of a warning message, a driver reacts by decelerating (with a regular deceleration rate, see Table 1), even if the brake light on the car ahead is not already lit.
- **C-RACCA:** This mechanism dynamically forms clusters of vehicles in the platoon. The first vehicle of a cluster is the Cluster Head (CH), which is in charge of relaying packets (e.g., emergency warning messages) from a CH in front to the rest of vehicles within the same cluster. This way, the number of redundant retransmissions is reduced.

Although the underlying message exchange mechanism is different, both procedures just make vehicles decelerate at a constant rate when they receive a warning message. Let us remark that our model is mainly concerned with the effects of the kinematic parameters and delays on accidents, unlike those proposals, whose goal is to control the communications broadcast storm. Since this control effectively reduces the warning message delay, it results in fewer accidents and so we can compare with our results.

In fact, our model is intended to be used from a different approach to the design of CCA applications: to quickly decide in which scenarios some parameters may have more influence on the collisions and so design the CCA based on it. As an example, in those proposals the main design goal of the communication system is to quickly deliver emergency messages, but according to our results in some scenarios a low delay is not relevant for the outcome, so we can design a CCA system that trades it off for additional reliability mechanisms.

Regarding the use of the model, the key step is to adequately define and model the input parameters. Kinematic parameters for different scenarios can be extracted from the literature, as is discussed in Sect. 5. Warning message delivery delay (latency) is one of the most difficult to model, since it depends on the transmission range of the nodes, the packet forwarding method used, the additional data traffic present in the channel, etc. In this case, we have modeled it by using the average latency between the reception of the message by two consecutive vehicles in the chain, since it was measured in [19] for different packet error rates. Nevertheless, it can be characterized in other ways, as for example by using the same average latency for all the vehicles in the chain, but this implies that all of them receive the message at the same time.

The kinematic parameters have been set equal to those used in [20] for both CCA methods, which are listed in Table 1. Once we have selected the parameters, we run the model 1000 times with different samples of the random parameters and extract the performance metrics of interest. Our model has been implemented with Matlab and it took only 8.65 seconds to run all the samples and extract results on a commodity PC with a quad-core processor at 2.2 GHz and 4 GB of memory.

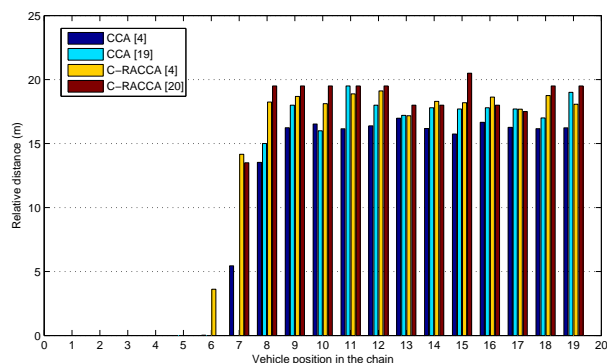
#### 4.1. Results

The results obtained by our model (tagged as [4]) for the performance metrics studied are presented in Fig. 3, together with the results provided by the authors of the two CCA mechanisms under consideration (tagged as [19, 20]). From the results of [20] we can only extract data for an inter-vehicle space of 15  $m$ , so we compute the average percentage of accidents only for this distance. For the basic CCA system we obtain 46.5% of accidents while the percentage in [20] is about 50%. On the other hand, for the C-RACCA scheme we obtain 40.2% of accidents, which is very close to the 40% showed in [20]. Both, the results of the model and the simulation show that C-RACCA outperforms the basic scheme due to its reduced delay.

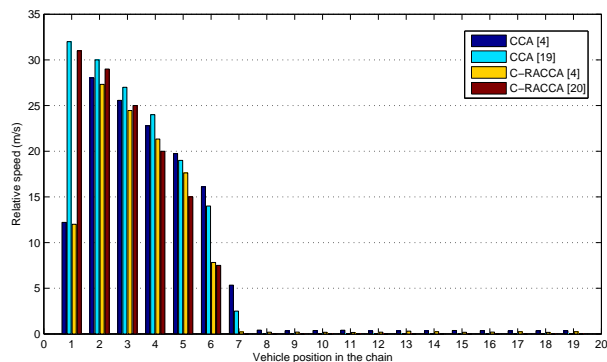
In addition, we compare with other metrics reported in [20]. For each vehicle in the chain, Figs. 3(a) and 3(b) show the average distance to the front car after

Table 1: Parameters for the CCA evaluation.

Number of vehicles	20
Vehicle speed	32 m/s
Inter-vehicle distance	15 m
Emergency deceleration	8 m/s <sup>2</sup>
Regular deceleration	4.9 m/s <sup>2</sup>
Driver's reaction time	$U(0.75, 1.5)$ s
Average relative delivery latency	CCA: 54 ms C-RACCA: 6.7 ms

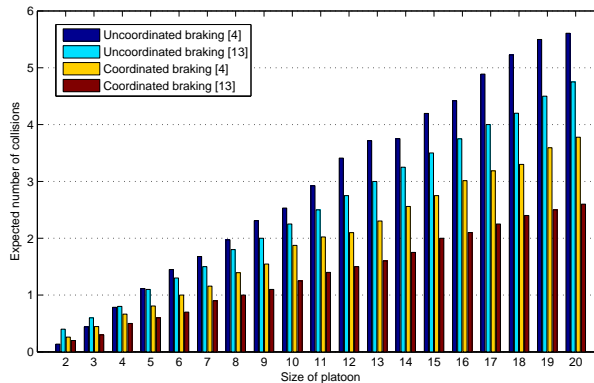


(a) Relative distance between two consecutive vehicles after stop.

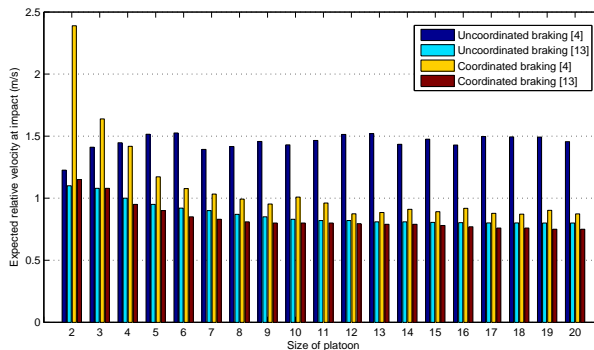


(b) Relative speed between two consecutive vehicles at the time of collision.

Figure 3: Performance metrics for the evaluation of CCA and C-RACCA mechanisms.



(a) Average number of collisions



(b) Relative speed between two consecutive vehicles at the time of collision.

Figure 4: Comparison with numerical results provided by Choi and Swaroop [13].

stopping and the average relative speed with respect to the front vehicle at the time of collision, respectively. Again, both the results of the model and the simulation coincide. The singularity of the first relative speed in the results comes from the assumption that vehicles stop instantly in case of collision. The first collision usually occurs when either the lead vehicle is braking but the follower is not, or both of them are braking. In both cases, the relative speed at the moment of the crash is not specially high. However, when they crash the speed drops immediately to zero. Thus, if there is a subsequent crash with the next vehicle, the relative speed is exactly the speed of the latter, which is usually high because it started braking recently. Looking at Fig. 5 in [19], this same trend can be seen. In Fig. 3(b) we show a comparison with the results reported by Fig. 12 in [20], which do not show this particularity. It seems that the lead vehicle also stops instantly but without further details in the original paper on how their simulation has been done we cannot discuss this discrepancy.

Despite slight differences, the results obtained by our model are in accordance with the results presented by the authors of the mechanisms, since our

numerical evaluation matches the simulation and shows that the C-RACCA approach reduces both the number of collisions and the impact of collisions when they inevitably occur. In this way, we have validated that our model is suitable for evaluating this kind of mechanisms.

In order to further validate our model, we compare it with a similar numerical tool previously proposed based on a stochastic model [13]. In Fig. 4 we provide a comparison of results provided by both of them. We have tried to faithfully reproduce their experiment by using constant parameters except for decelerations, but our models differ in several aspects, discussed in Section 2. In spite of these differences both models show coincident results. Our model is more pessimistic, as expected since it provides an upper bound, whereas theirs provides a lower bound, but the trend is similar. Our model also shows the apparent anomaly of platoons of size 2, discussed in [13].

## 5. Design of CCA applications

In this section we exemplify the use of our model for CCA design by evaluating two different scenarios under different traffic conditions. It is important to remark that our results provide broad directions for design at an early stage which would typically be refined in later stages. Our goal is to derive mainly qualitative conclusions about the importance of each kinematic parameter in the development of a CCA application. The first step is to define the scenarios and to model adequately the different input parameters. This step is key in the quality of the results and it requires a research effort from open literature to properly characterize the model parameters. Next, some input parameters are set according to hypothetical CCA systems and performance metrics are computed for the different scenarios and compared. For instance, we can test if a hypothetical CCA system able to just remove reaction times reduces the number of accidents. Finally, we evaluate CCA mechanisms which could control several parameters simultaneously in order to improve the traffic safety.

### 5.1. Scenario and parameter characterization

Let us recall that our model needs that the five input parameters are characterized, number of vehicles by a fixed value, inter-vehicle spacing by a random distribution and speeds, delays and decelerations by deterministic or random values. The first scenario we consider is the freeway studied in [27], where three different time periods with different traffic flows were observed and characterized:

1. Night traffic: very low traffic density and high speed;
2. Free-flow traffic: moderate traffic density and high speed;
3. Rush-hour traffic: very high traffic density and low speed.

Wisitpongphan *et al.* [27] showed that during the night period the inter-vehicle spacing can be modeled by an exponential distribution, while during the



Table 2: Probability distributions for the vehicle speed and inter-vehicle distance.  $N$ ,  $EXP$ ,  $LN$  and  $LL$  represent the normal, exponential, log-normal and log-logistic distributions respectively.

Scenario		Vehicle speed	Inter-vehicle distance
Freeway	Night	$N(30.93, 1.2) \text{ m/s}$	$EXP(256.41) \text{ m}$
	Free-flow	$N(29.15, 1.5) \text{ m/s}$	$LN(3.4, 0.75) \text{ m}$
	Rush-hour	$N(10.73, 2) \text{ m/s}$	$LN(2.5, 0.5) \text{ m}$
Urban	Peak hours	$N(6.083, 1.2) \text{ m/s}$	$LL(1.096, 0.314) \text{ m}$
	Non-peak hours	$N(12.86, 1.5) \text{ m/s}$	$LN(0.685, 0.618) \text{ m}$

other time periods the log-normal distribution<sup>3</sup> provides a better fit. Moreover, they showed that regardless of the time of day, the speed of vehicles follows a normal distribution. The probability distributions used for these parameters are given in Table 2.

The second scenario we consider is the urban scenario studied in [28], where two different time periods were considered:

1. Peak hours: the traffic is in congestion status;
2. Non-peak hours: the traffic is in free-flow status.

Yin *et al.* [28] showed that the headway data (and so the inter-vehicle spacing) can be modeled by the log-normal distribution for peak hours and by the log-logistic distribution<sup>4</sup> for non-peak hours data. Table 2 shows the parameters used for these probability distributions.

At this point we have the input data for the inter-vehicle distance for all the scenarios and time periods, as summarized in Table 2. We use other references to extract the rest of the parameters needed by our model. Let us recall also that delay  $\delta_i = T_{r,i} + T_{m,i}$  is actually the sum of message transmission time (latency) and driver reaction time, so we need to characterize both of them.

- **Driver's reaction time.** Taoka [29] estimated the distribution of the driver reaction time by fitting a lognormal distribution to the data collected by Michael Sivak, used also by [5]. We use then a lognormal distribution with mean 1.21 s and standard deviation 0.63 s for the driver reaction time,  $T_{r,i}$ .

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<sup>3</sup>The probability density function of a log-normal distribution is:  $f(x; \mu, \sigma) = \frac{1}{x\sigma\sqrt{2\pi}} e^{-\frac{(\ln x - \mu)^2}{2\sigma^2}}$ ,  $x > 0$ .

<sup>4</sup>The probability density function of a log-logistic distribution is:  $f(x; \mu, \sigma) = \frac{e^{-\frac{\ln x - \mu}{\sigma}}}{\sigma x \left(1 + e^{-\frac{\ln x - \mu}{\sigma}}\right)^2}$ ,  $x > 0$ .

- **Warning message delivery latency.** According to Fracchia and Meo [30], the average time needed by the warning message (using a probabilistic broadcasting scheme) to reach the farthest node in an area of 2 Km is always under 0.1 s. So, we will use this value for the message latency,  $T_{m,i}$  as an upper bound.
- **Deceleration.** We assume that the first vehicle decelerates at  $16 m/s^2$  (which models a vehicle colliding with a fixed obstacle and stopping within a very short distance). The rest of vehicles will decelerate at their maximum braking capabilities. In [5] the authors show that this braking capability (considering light passenger vehicles and dry pavement) fits a normal distribution with mean  $7.01 m/s^2$  and standard deviation  $1.01 m/s^2$ .

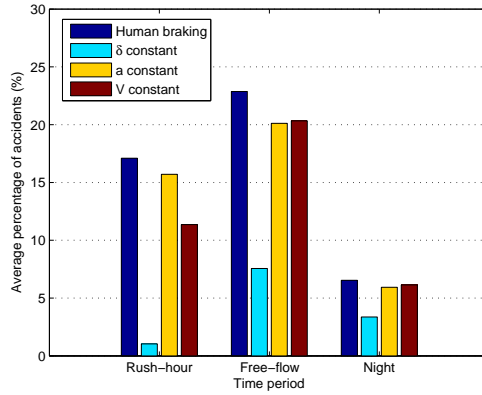
Therefore, during the model initialization we fill the input vectors of speeds, decelerations and delays with a random sample of the appropriate distribution. As we discussed in Sect. 3.3, setting pure random values to all the parameters results in unrealistic values. Fortunately, it can be corrected by adjusting the values during initialization. Therefore, we truncate all the normal distributions within a sufficiently wide range, for instance deceleration is kept between  $5.5 m/s^2$  and  $8.5 m/s^2$ . In addition, when setting the value of consecutive vehicles in the input vector of speeds and decelerations, we do not allow the relative speeds and the relative decelerations to exceed a certain threshold. This way we keep the parameters essentially random but avoid unrealistic situations in both the initial state, like a vehicle driving much faster than the preceding one, and the braking process, like starting to decelerate too late and soft.

### 5.2. Influence of isolated controlled parameters

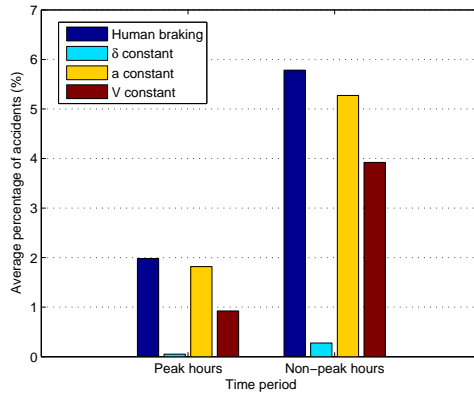
In this subsection we evaluate the influence of each one of the parameters independently of the average number of accidents. The goal is to evaluate the effects of hypothetical CCA systems on the number of accidents. A basic CCA warning message system is always assumed to be in place, since it is the main assumption of the model, so vehicles are notified when the incident occurs. Additionally, a reactive CCA application may control delays, with the communication system, and decelerations, with some automated control response to the warning message. Assuming those systems are used allows to set arbitrary values to them, instead of random variables. In order to assign deterministic values to speeds it would require to assume the use of a proactive CCA system, that is, a system that keeps the relative speeds of vehicles within a certain range at any time.

To test these possibilities, in Fig. 5 we compare the average percentage of accidents that occur in a chain of 20 vehicles driving in the described scenarios with the following conditions:

- **Human braking:** the drivers are only informed about the emergency situation and they react on their own. Reaction times, speeds and decelerations are random to some extent, since they are human-controlled,



(a) Freeway scenario

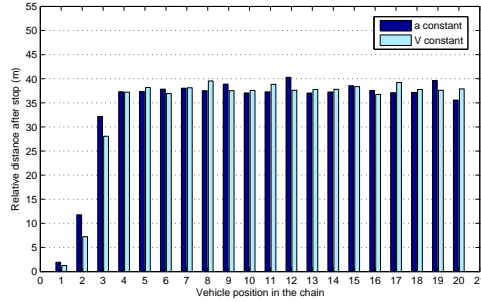


(b) Urban scenario

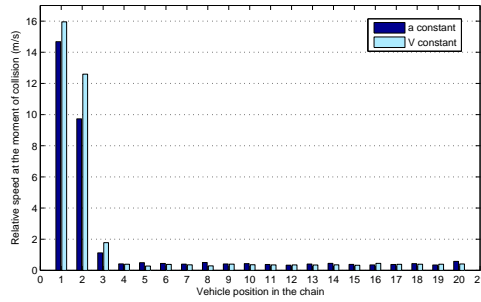
Figure 5: Influence of the principal kinematic parameters on the average percentage of accidents for the scenarios under evaluation.

which means that all the parameters are assigned the random distributions specified in the previous subsection.

- **$\delta$  constant:** a CCA system is able to automatically start braking, so delay is fixed and equal to  $0.1\text{ s}$  for all the vehicles in the chain. The rest of the parameters follow the distributions specified in the previous subsection. It means that decelerations are still subject to random variation.
- **$a$  constant:** a CCA system makes all decelerations be equal to  $8\text{ m/s}^2$  for all the vehicles in the chain. The rest of the parameters follow the distributions specified in the previous subsection. In this case, random driver reaction time is present.



(a) Relative distance after stop for each vehicle in the chain.



(b) Relative speed at the moment of collision for each vehicle in the chain.

Figure 6: Influence of the deceleration rate and the vehicle speed on the relative distance and the relative speed for Free-flow traffic in the Freeway scenario.

- **V constant:** a proactive CCA system makes the speed constant and equal to the average speed of the scenario for all the vehicles in the chain. The rest of the parameters follow the distributions specified in the previous subsection.

Let us note that fixing delays and decelerations independently is somehow unrealistic. A delay system able to automatically start braking, thus removing the driver reaction time, should brake in a controlled way. However, we are not concerned at this point with the feasibility of these systems, but just evaluate the effect of controlling each one of the principal kinematic parameters.

In all the cases the most significant reduction in the percentage of accidents is obtained by controlling the delays. Therefore, the main goal of a CCA application should be to remove the variability of the drivers' reaction time and make cars start braking simultaneously. Hence, although a warning message may help reduce the number of accidents compared to no CCA application at all, to be really effective it needs additional measures involving taking over driver control. In fact, the message delay is not actually relevant to the outcome since it is even in the worst cases, one order of magnitude lower than the driver reaction

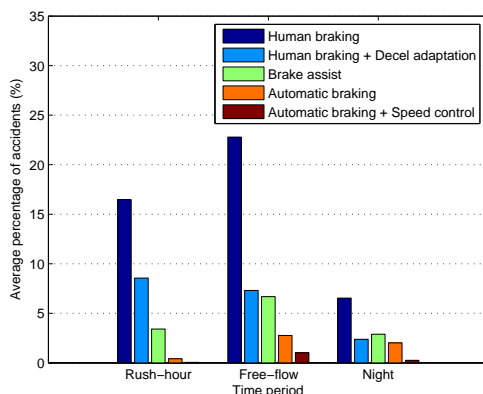
time. Additionally, for low and medium speeds it seems to be more necessary to control the speed than the deceleration rate.

On the other hand, in high speed scenarios, controlling the deceleration rate and the speed result in a similar reduction in the percentage of accidents. Besides, in Fig. 6 we can observe that for the Free-flow traffic in the Freeway scenario (the case of Night traffic results in a similar performance) controlling the deceleration rate, the metrics of relative distance and relative speed show a slightly superior performance than controlling the speed, but there is not a significant improvement.

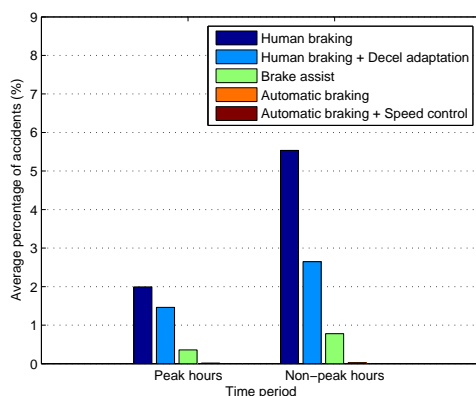
### 5.3. Influence of combined controlled parameters

In the previous section we evaluated the influence of controlling some parameters independently of others. In this subsection, we evaluate “more realistic” CCA systems based on the results obtained previously, where the system would be able to control several parameters simultaneously as follows:

- **Automatic braking.** A reactive CCA mechanism that allows the automatic braking of the vehicles in the chain, that is, when the vehicle receives the warning message it immediately starts to brake, although the driver is not yet aware of the risk. In this case we can assume that the delay of braking and the deceleration rate are controlled (fixed) by the application, whereas the rest of the parameters follow the distributions specified previously.
- **Automatic braking + Speed control.** A proactive CCA mechanism that controls the speed of vehicles before the emergency situation, in addition to allowing the automatic braking of the vehicles in the chain. In this case delays and decelerations are constant and equal to  $\delta = 0.1 s$ ,  $a = 8 m/s^2$ , whereas the inter-vehicle spacing follows the distribution specified in the scenario description and speed is constant and equal to the mean of the distribution.
- **Human braking + Deceleration adaptation.** In the previous subsection we described the “Human braking” behavior. Now we assume a more realistic situation in which the driver adapts its deceleration rate to the velocity and deceleration of the preceding vehicle. In Sect. 5.1 we mentioned that to avoid unrealistic results, the random samples are forced to be within certain ranges. Here we additionally assume a rational deceleration, that is, when the initial deceleration vectors are filled it is checked that the random sample is within the range, but also that the relative values are reasonable. For example, if the preceding vehicle is braking at a high rate, it is not realistic that the follower brakes softer and let itself collide, so its deceleration is set to a higher value, but not above the maximum deceleration. Or if relative speeds seem to allow a safety stop, it is set to a comfortable deceleration.



(a) Freeway scenario



(b) Urban scenario

Figure 7: Average percentage of accidents for different CCA proposals in the scenarios under evaluation.

- Brake assist.** A mixture of human braking and automatic braking. If the driver has not reacted and started to brake (with rational deceleration) after a given time threshold, the system automatically starts braking at a constant maximum deceleration. The threshold has been set to  $0.84\text{ s}$  which is the mode of the lognormal distribution provided in [29]. This case models the behavior of current brake assistance systems, and possibly the most likely CCA to be deployed in the near future.

Fig. 7 shows the average percentage of accidents that occur in a chain of 20 vehicles driving in the described scenarios when these types of CCA mechanisms are in operation as well as both types of human braking. Since our model provides upper bounds on the number of accidents, instead of focusing

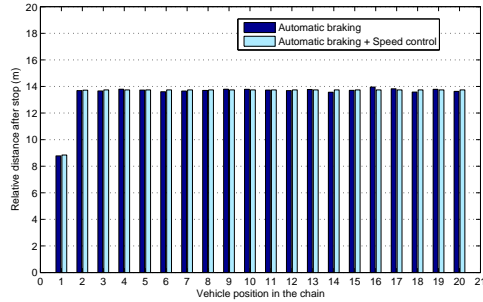
on the particular absolute values it is possibly more useful to compare qualitatively different systems. Looking at the figure, it can be noticed that the percentage of accidents in the case of rational deceleration adaptation is much lower, more than 50%, than when it is not considered. But even considering this human braking behavior, more sophisticated CCA applications still reduce the percentage of accidents. In this sense, our results agree with previous studies [5, 13], as expected.

Brake assist, as we said, models approximately current systems and is the most likely CCA to be deployed in the near future. Its performance is between rational human and automatic (coordinated) braking as should also be expected, but still much closer to human braking, showing that there are still potential gains from enabling systems with more cooperation, as automatic braking. In fact, these gains are dependent on the reaction threshold, which should be carefully selected.

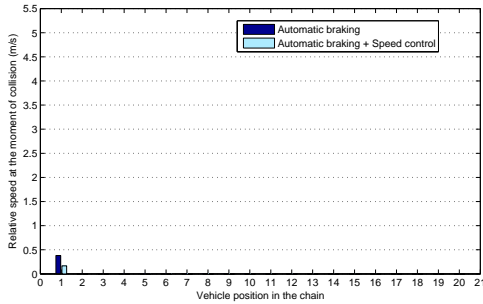
In high speed scenarios (Free-flow and Night period in the Freeway scenario), automatic braking, i.e. coordinated deceleration, effectively reduces the number of accidents with respect to human braking variants, even up to 50% of a Brake assist system in Free-flow. There would be a remarkable reduction in the percentage of accidents if we could employ speed control in addition to automatic braking, but it would require a much more coordinated type of AHS, which would be close to the Platooned Vehicles concept discussed in [5]. Development of such a concept is still technically challenging, whereas implementation of coordinated braking policies seems to be more likely in the near future. Speed control benefits are particularly clear in the Night scenario, where large inter-vehicle spacing makes performance of all the variations of deceleration control practically equal, whereas control of speed would actually reduce the accidents.

However, in low and medium speed scenarios (Urban scenario and Rush-hour in the Freeway scenario), there are no significant differences between the more coordinated CCA policies (automatic and speed control). Besides, if we observe Fig. 8 it can be seen that for the case of Rush-hour in the Freeway scenario (the cases in Urban scenario result in a similar performance), the severity of accidents and the margin of safety are very similar as well. In fact, in these scenarios Brake assist is able to remove most of the collisions.

According to these results, it would seem sound to focus on the design of a cooperative CCA system which is able to notify about incidents and, in case of emergency, apply a coordinated braking policy according to the known status of the surrounding vehicles. A less ambitious system based mainly on control of delays, as the Brake assist model can bring relevant improvements in most of the scenarios considered. On the contrary, a cooperative system to control speeds poses much more technical challenges and its cost-benefit ratio is not so evident at a first approach. As a summary of this section, we have exemplified how to use our model to draw qualitative and quantitative results about the influence of the different parameters for the design of CCA applications and have described the key steps.



(a) Relative distance after stop for each vehicle in the chain.



(b) Relative speed at the moment of collision for each vehicle in the chain.

Figure 8: Comparison of the relative distance and the relative speed between the two CCA proposals for Rush-hour traffic in the Freeway scenario.

## 6. Conclusions

In this paper we have shown how CCA mechanisms can be evaluated numerically by using a stochastic model as an effective alternative to simulation. We have discussed the limitations of current traffic simulators for accident simulation and introduced our stochastic model. The model was derived in a previous paper and here we discuss its limitations, validate its results against available previous results and provide and discuss abundant application examples. The main limitation of the model in its current form is that independence between state input variables, relative velocity and inter-vehicle spacing, is assumed, which is not realistic in many cases and introduces too much randomness leading to pessimistic results. It can partially be corrected by adjusting the input variables and their relative values, as we have discussed.

We have illustrated its capabilities as an assessment tool for CCA application design while describing the working methodology. To this purpose, we have evaluated different types of CCA applications in two scenarios, a freeway and an urban scenario. The results suggest that the variability due to the drivers reaction time is the main cause of accidents and so removing it should be the



main focus of a CCA application. This could be possible by automatic braking, that is, when the vehicle receives the warning message it takes over control and immediately starts to apply a coordinated braking policy, even though the driver is not yet aware of the risk. This is one of the different CCA systems discussed. Results suggest that the benefits of implementing this CCA are relevant. On the contrary, results show that the benefits of implementing a much more challenging cooperative system, able to coordinate speeds, are marginal in most of the cases. In any case, our main goal has been to show the model potential as an aiding design tool rather than proposing a particular CCA system.

As future work, we intend to enhance the model in order to deal with the mentioned limitations. So, the first step is to introduce bivariate distributions in the model, to capture state variable correlations, increasing the model accuracy. As a second step it would be necessary to find appropriate joint distributions for speed and inter-vehicle spacing. There is actually a lack of empirical models that jointly describe inter-vehicle spacing and speed. Similarly, we have shown how to characterize the input variable distributions by using statistical models proposed in the open literature, but additional efforts in the empirical characterization of deceleration, reaction times and communication delays are clearly necessary.

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