An analytical approach to the optimal deployment of Wireless Sensor Networks

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Summary. In this work we propose and investigate a novel research topic in Wireless Sensor Networks (WSNs): sensor deployment in order to maximize the interest of the gathered information. The target areas are characterized by zones with different interest levels. We model the interest variable as an "importance function" that assigns to each point a quantitative reference. Due to the nature of WSNs, the sensor deployment must guarantee that the information of all nodes can reach certain control nodes or *sinks*. Otherwise, the events captured by the nodes are lost. This condition is equivalent to ensuring the existence of a communication path from every node to at least one of the sinks. We propose a global optimization model that fulfills all the conditions in the model and solve it with a simulated annealing algorithm that determines optimal node placement. Moreover, we also characterize the effects of an aerial deployment, that is, the effect of node placement inaccuracy in network behavior and performance. We have found that small placement variations may lead to strong fluctuations in the quality of the connectivity properties of the network topology, and thus to a significant performance degradation in terms of captured interest.

1 Introduction

The thriving recent research on Wireless Sensor Networks (WSN) is due to a great extent to the potential benefits for a broad range of applications. Surveillance, environment monitoring and biological risks detection are among them [1]. So far, many aspects of WSN have received research attention, particularly those related to networking issues and self-organization.

Besides, the quality of the sensing procedure and data acquisition is crucial for the network to efficiently perform its intended task, and has to be considered at an early design stage. Sensing quality depends on the *coverage* of the network. In a wide sense, this issue comprises many aspects regarding different properties of the network. In fact, in sensor nodes, the term "coverage" may refer to different concepts, for instance,

- *Wireless coverage* is the area around a node were it can effectively communicate.
- *Sensing coverage* is the area around a node were it can effectively sense some physical magnitude.

The former is related to the connectivity properties of the network and a variety of associated problems, being connectivity a constraint in most of them, i.e. minimizing some parameter of interest as energy consumption. The latter determines the quality of the sensed information in general, although this receives many interpretations, and opens another broad set research of problems [2].

In this paper, we consider a particular class of sensing coverage problems, defined in [5] as *constrained coverage* problems, that is, finding a deployment configuration that maximizes the collective sensing coverage of the nodes, while satisfying one or more constraints. In this work there are two main differences with previous works, yielding a more realistic scenario:

1. Some zones in the target area are more "interesting" than others, unlike in previous models, which assume an *uniformly distributed* interest across the target area. In other words, we consider that some distribution function of the target area describes the probability of sensing (capturing) an event of interest.
2. We also consider a network deployment stage that introduces placement inaccuracies, and how it affects the global sensing capacity of the network.

Martian exploration helps us illustrate our discussion. So far, Martian probes and rovers have only been able to explore small areas. Which are the "interest areas" in this context? The best chances of finding traces of life byproducts seem to be linked to the past presence of water. Thus, a probe should look in terrains whose composition and properties suggest such presence.

Current spacecrafts are bulky because they must carry engine fuel, power supply systems for the high gain satellite link antennas, and so on. Wireless

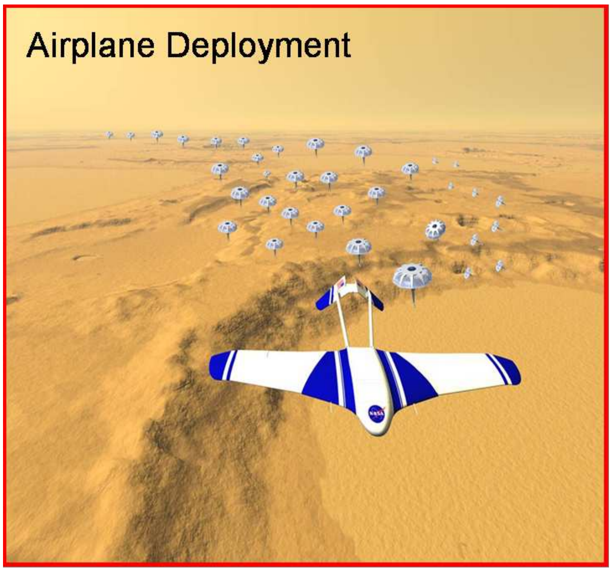

Fig. 1. Aerial sensor deployment over Mars surface (artist conception, credit: NASA/JPL)

Sensor Networks may become a feasible alternative to extend their monitoring range, by deploying large quantities of sensors over the interest areas (see Fig. 1). The interest area is defined by the probability of occurrence of some event of interest (in Fig. 2, the colors of the areas indicate the probabilities of the interest events). The question we face then is: how to spread the sensors over those areas to maximize the probability of capturing events of interest? This is the *sensing coverage goal*, for a (possibly) non-uniform distribution of the probability of events. In addition, since sensor nodes are limited devices, they must report the captured events to a special node, the *sink*. In the Martian exploration example, only the probe is able to set a long distance satellite link. Thus, the information must hop from sensor to sensor towards the probe. Therefore, the location of sensors must allow a global communication with the sink. If a node or a group of them are isolated, their events will be *lost*. This leads us to a constrained coverage problem, since a connected network is compulsory.

Fig. 3 illustrates the problem. Every sensor has a sensing range (r_s) and a communications range (r_t). The distance between hops must be less than s_r and node location should allow to capture as many events as possible subject to r_s. Now, two possibilities can be considered to deploy the sensing nodes:

- Deterministic: we assume that sink and sensor position can be individual and precisely selected. The solution to the constrained coverage problem provides the set of node locations. If we consider an aircraft deployment, it is unlikely that the nodes will precisely fall in their selected optimal

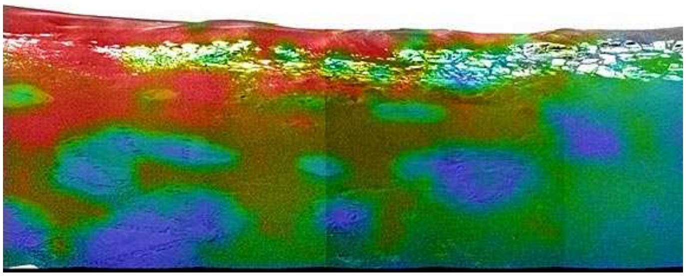

Fig. 2. NASA Opportunity Mars Rover image: MiniTES hematite measurements taken by the thermal emission spectrometer superimposed on an image from the panoramic camera. Credit: NASA/JPL

points, and thus the influence of actual sensor positions on performance metric must be studied.

- Random: we assume that the position of the probe can be precisely selected, but the nodes are randomly spread around it (the probe itself launches the nodes, which is called a *ballistic surface deployment*). In this case, the problem is much harder. The results of the optimization problem should provide the parameters that aim the shots (like elevation and speed).

In this work we focus on the *deterministic* case. We introduce a mathematical programming formulation where the objective function seeks to maximize the collective interest, and the constraints reflect the need of a connected network. The rest of this chapter is organized as follows: background is discussed in section 2. Section 3 provides a formal description of the problem, as well as the parameters and variables involved. Section 4 describes the objective function of a global optimization approach to the problem. Using it, section 5 presents results for two synthetic scenarios: with large convex areas and small non-convex areas, respectively. Then, section 6 studies the effect of random variations in the position of the nodes, to characterize aerial deployment. Finally, section 7 discusses the main conclusions of this paper.

2 Related work

As mentioned in the introduction WSN *coverage* refers to wireless and sensing range. Concerning the first, there is plenty of research on the influence of a variety of parameters in the network. In particular, the transmission power needed to guarantee connectivity is discussed in [8]. Other aspects like energy consumption or node failures have also been considered [9, 10]. In addition, diverse protocols that guarantee network connectivity when the nodes fail or sleep have been proposed [11, 12]. None of these works consider sensing coverage.

On the contrary, in [13] the relationship between sensing coverage and connectivity is discussed. The concept of *K-coverage* is also introduced: a region is K-covered if every location inside it is at least covered by K nodes. It is shown that, if $r_t \geq 2r_s$, 1-coverage implies connectivity. The authors further propose a Coverage Configuration Protocol (CCP), which configures the network to dynamically provide the required degree of coverage (these protocols are called *density control protocols*. The work in [6] is similar, including a density control protocol that considers energy consumption to maximize the network lifetime.

In [7], the problem of achieving K-coverage while conserving the maximum energy is addressed. The authors find the number of sensors that provides K-coverage degree for grid and random deployments. In those cases, it is assumed that all points have the same importance within the area of interest. Interestingly, in our work we model the importance of a point as a probability density function, and we find the locations that maximize such probability. Note that we do not employ the concept of K-coverage. In fact, our optimization model avoids overlapping of sensing areas. This approach is useful for applications requiring several sensor readings of an area, as in vehicle tracking. Moreover, we introduce the concept of *K-survivability*, that is, we obtain the optimal locations that ensure connectivity in case of K sensor failures.

Regarding sensor deployment, reference [5] provides a self-deployment strategy for robots than move to optimal positions. The study in [14] is similar to ours. In it, the authors discuss the inherent uncertainty of sensor readings and assign a probability to sensor detections. These probabilities help to model terrain effects. The authors also describe an algorithm to place the sensors in a grid in order to provide adequate coverage. Instead, our algorithm selects positions in a continuous space.

A similar problem recently studied in WSN is optimal positioning of base station (sink) [15]. Extending network lifetime or increasing responsiveness of data delivery have been used as performance metrics to obtain the optimal base station positioning. Static and dynamic techniques for positioning have been proposed. This work differs in both goal (maximizing the probability of capturing events) and domain of application (we look for the optimal position of a group of sensors, one of them being also a sink).

Finally, the issue of aerial sensor seeding has been covered in [16]. The algorithms in that work do not look for sensor positions, but control the movement of a flying robot to drop the sensors according to a planned topology. Thus, our proposal is complementary, in the sense that our results could be used as an input for the seeding robot.

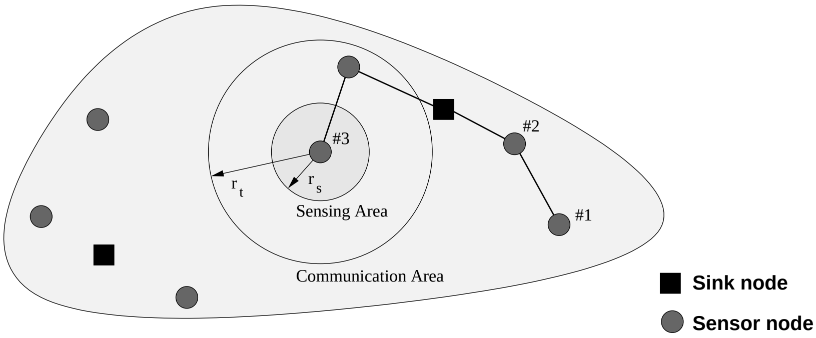

Fig. 3. Network deployment

3 Scenario and problem description

In this section we describe our constrained coverage problem. As stated in the introduction, we aim at selecting the position of N sensors in a target area, by maximizing the probability of capturing events, or equivalently, by maximizing the interest (or importance) of the covered area. Henceforth, let us assume a Bi-dimensional target area. Let us characterize our model by defining the following parameters:

- The interest of a position is defined by means of an *importance function* $\alpha(x)$, which maps each position x onto a value that is indicative of its importance.
- The total number of nodes (including sink nodes), N.
- A *transmission range* r_t, which is the maximum distance for two sensor transmitters to be mutually visible.
- A *sensing range* r_s, which determines the region where a sensor can collect measures.

The following considerations also hold:

- Sink nodes also have sensing capabilities, like conventional ones.
- For simplicity, in each scenario there is a single sink node.
- The sensing range is much lower than the transmission range ($r_s << r_t$).
- The importance function around a point x has small variations. Therefore, $\alpha(x^*) \simeq \alpha(x)$, if x is close enough to $x*$.
- The topology of the scenario is known. More formally, all points x belong to a well defined *compact* set $Z \subset \mathbb{R}^2$.

Given any two points $x_1, x_2 \in Z$ let us denote $\delta(x_1, x_2) = ||x_1 - x_2||^2$ as the *euclidean* distance between points x_1 and x_2. Let $S_i = \{x^* \in Z : \delta(x^*, x_i) < r_s\}$, $x_i \in X$ denote the open ball of center x_i and radius r_s. The joint importance captured by the N nodes in positions $(x_1, \ldots, x_N)$ is given by:

$$\begin{aligned} \alpha &= \int_{\forall x \in S} \alpha(x) \\ \text{being } S &= \bigcup_{i=1 \ldots N} S_i \end{aligned} \quad (1)$$

This can be simplified if we consider that overlapping of sensing areas of neighbor sensors is unlikely. Thus, we obtain:

$$\alpha = \sum_{i=1}^{N} \int_{\forall x \in S_i} \alpha(x) \quad (2)$$

Let us simply denote $\alpha(x_i)$ as α_i. A further simplification is possible. If, in the sensing area of a sensor, the importance function has small variations[3], then equation (2) can be rewritten as:

$$\alpha = \sum_{i=1}^{N} \pi r_s^2 \alpha_i \quad (3)$$

In fact, since πr_s^2 is constant, the problem is equivalent to maximizing:

$$\hat{\alpha} = \sum_{i=1}^{N} \alpha_i \quad (4)$$

That is, selecting the points in the interest area where the event probability is maximum, while satisfying that there exists at least one route from every sensor to the sink, even in case of node failures.

The resulting problem has local minima, and the objective function is not continuous due to the influence of the terrain (irregularly shaped regions of interest, line-of-sight obstacles) and the aforementioned connectivity constraint in case of errors. Therefore, it may be advisable to reformulate it as a global optimization problem (by taking the constraints to the objective function). Then it can be solved with proper heuristics. This approach is developed in the next section.

4 Optimization model

Let us consider a synthetic area $Z \in \mathbb{R}^2$ with regions of interest, like the one sketched in figure 5. Clearly, in large/complex scenarios manual placement is

[3]Let us remark that this does not imply that the importance function is uniform in the entire monitored area, but that the probability is *locally* uniform (around the node), which is reasonable if the physical magnitude to be sensed is continuous.

not possible, specially when only a small number of sensors is available. Thus, we are interested in proposing a practical optimization model that approximates the solution of the problem we formulated in section 3.

As stated in the previous section, we want to deploy N sensors in a set of locations $X = (x_1, \ldots, x_N)$, $x_i \in Z$. Let α_i be the given importance of location x_i. Additionally, let β be a penalty on sensor coverage overlapping. We define the following quality functions:

- $f_{min}(X) : \mathbb{R}^{2N} \rightarrow \mathbb{R}$, $f_{min}((X \setminus \{x_i\}) \cup \{x_i + d\}) = \min(\delta(x_i + d, x_j))$, $x_i, x_j \in X$, $\forall i \neq j$. It measures the minimum separation between a given sensor whose position is shifted d units in $\mathbb{R}^2$ and all the others, and
- $f_s(X) : \mathbb{R}^{2N} \rightarrow \mathbb{R}$, $f_s(X) = \sum_{i=1}^{N} \alpha_i \cdot \text{area}(S_i) - \beta \cdot \text{area} \bigcap_{i=1}^{N} S_i$. It represents the joint importance of the measurements of the sensors, and penalizes coverage overlapping.

Our objective function is:

$$f^K(X) = \begin{cases} \gamma \cdot f_{min}(X) + f_s(X) & \text{if } X \text{ is } K\text{-survivable} \\ C & \text{otherwise,} \end{cases}$$

where C is a suitable constant. Set X is said to be K-survivable if, for any simultaneous failure of K sensors in X, every surviving sensor in x_i lies within the transmission coverage of another surviving sensor x_j ($\delta(x_i, x_j) \leq r_t$) at least, and every surviving sensor must communicate with a sink, either directly (single hop) or indirectly (multiple hops). It is assumed that at least one location in X with guaranteed survivability corresponds to a sink.

We define the *sensor placement optimization problem* as:

$$\begin{aligned} & \underset{X}{maximize} \ f^K(X) \\ & subject\ to : \{x_1, \ldots, x_N\} \in Z, \end{aligned} \tag{5}$$

which satisfies the following conditions:

A1. It is a box-constrained global optimization problem.
A2. $f^K(X) : \mathbb{R}^{2N} \rightarrow \mathbb{R}$ is upper bounded.
A3. $f(X)$ has directional derivatives $f'(X, d_i)$ everywhere defined along the canonical directions and their opposite (which positively span R^{2N}), and

$$\eta > 0 \Rightarrow \begin{cases} f'(X, \eta d_i) = \eta f'(X, d_i), \\ f(X + \eta d_i) = f(X) + \eta f'(X, d_i) + o(\eta) \end{cases} \tag{6}$$

Consequently, it is possible to apply the derivative-free method in [17], which converges to a stationary point of $f^K(X)$ regarding the directions given by the canonical base and its opposite. Alternatively, it is possible to approximate the solution with heuristics such as neighborhood search or simulated annealing. We have chosen the latter for the preliminary tests in this paper.

(a) Convex topology (b) Non-convex topology

Fig. 4. Scenarios in the numerical test

5 Numerical tests

We generated two synthetic scenarios: "convex/large regions" and "non-convex/small" regions to test the performance of the optimization algorithm in the previous section. Figure 4 illustrates them. In order to assign importance levels, we cover each scenario with a 640×480 units bitmap. Each bit has a weight from 0 (minimum interest) to 255 (maximum). Figure 4 represents it with a color scale (light→low interest, dark→high). To determine the importance of a sensor placement, we read the weight of the surrounding pixels. In the scenarios we spread $N = 30$ sensors with a coverage radius of $r_s = 10$ units and a transmission range of $r_t = 60$ units.

The initial location of all sensors is the center of the scenario x_c, in order to guarantee that we depart from a K-survivable set X. In order to speed up the calculations up we do not compute $f_s(X_n)$ exactly, but approximate it as follows:

a) Set $f_s(X_n) = 0.0$
b) for $i = 1, \ldots, N$
   i) $f_s(X_n) = f_s(X_n) + \alpha_i S_i$
   ii) for $j = i + 1, \ldots, N$
      if $\delta(x_i, x_j) < 2r_s$ $f_s(X_n) = f_s(X_n) - (4r_s^2 - \delta(x_i, x_j)^2)$

The set X_n is K-survivable if every subset that results from extracting K locations from X_n is 0-survivable. To check K-survivability we proceed as follows:

We define three tables, *original*, *current* and *next*. It is assumed that sinks always survive. We assign all surviving nodes (simply *nodes*, hereafter) but the sinks to *original*. We assign all the sinks to *current*. Then we move all the nodes that fall within the coverage of a sink to *next*.

At that point, every node with direct communication with a sink belongs to *next*. The nodes in *original* may communicate with the nodes in *current* through some nodes in *next*.

As a last step, we delete all the nodes in *current*, move the nodes in *next* to *current* and repeat the entire process until *original* is empty (the set is survivable) or it is impossible to move any node from *original* to *next* (the set is not survivable).

We are now ready to define our simulated annealing algorithm:

Simulated annealing algorithm:

Initialization: $T = T_0$, $i = 0$. Set $x_i = x_c$, $i = 1, \ldots, N$.
While $T > 0$:

a) $i = i + 1$. If $i > N$, $i = 1$ and $T = T - \triangle T$.
b) Let $v \in I\!R^2$ be a random direction such that $||v|| = 1$. Set $d = T \cdot r_t \cdot v$.
c) If $x_i + d \notin Z$ or $X^* = X - \{x_i\} + \{x_i + d\}$ is not K-survivable, goto a).
d) If $f^K(X^*) > f^K(X)$, $p = 1 - 0.5T$. Otherwise, $p = 0.5T$.
e) If $rand() < p$, $X = X^*$.

Constant C is implicit, in the sense that it is a lower bound for $f^K(X)$ that prevents from updating the set of locations. The other settings in our tests were $T_0 = 1$ and $\triangle T = 10^{-4}$. Note the importance of parameter γ in the definition of $f^K(X)$. A small value makes difficult the colonization of new regions of interest, since the nodes try to stick together as far as the value of $f_s(X)$ is large. A large value tends to disperse the nodes regardless of the importance of the regions they measure. Therefore, it is necessary to tune this parameter. We started from $\gamma = 0$ and kept increasing it in 0.1 steps until the nodes began to ignore the regions of interest.

Figures 5 to 8 show our results for $K = 0$ and $K = 1$. In each case we indicate the value of γ after tuning it (we started with $\gamma = 0$ and increased it in 0.1 steps. We chose the largest value that produced a non-degenerate case, which we identified visually). Elapsed times on a Pentium IV were under 10 seconds for $K = 0$ and less than 50 seconds for $K = 1$. It is interesting to observe that setting $K = 1$ produces alternative paths, but the algorithm tends to use as few nodes as possible to create "bridges" between regions of interest.

6 Characterization of aerial deployment

To complement the results in the previous sections, we also model the impact of the inaccuracies of aerial deployment. We approximate them as random changes that determine the final position of nodes. Their effect is twofold. First, the captured importance may be lower in the real positions. Second, and

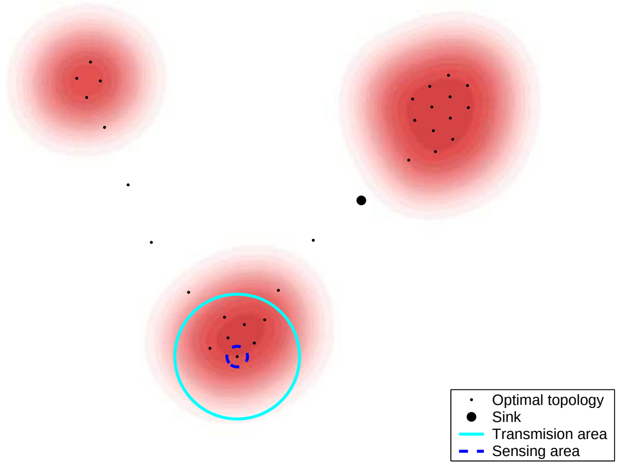

Fig. 5. Convex/large regions scenario, $K = 0$, $\gamma = 1.0$

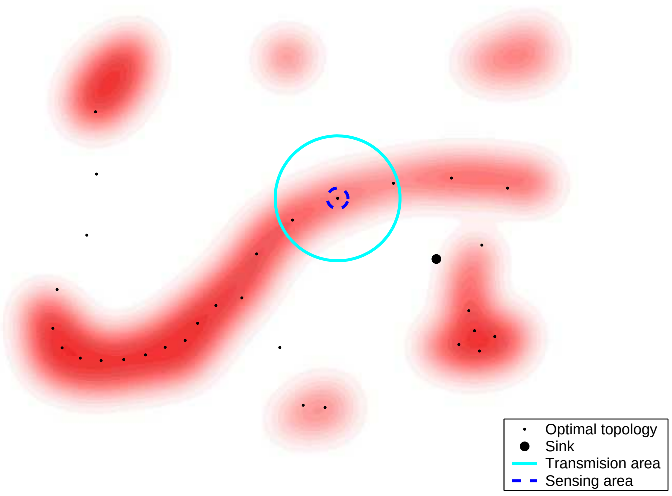

Fig. 6. Non-convex/small regions scenario, $K = 0$, $\gamma = 1.1$

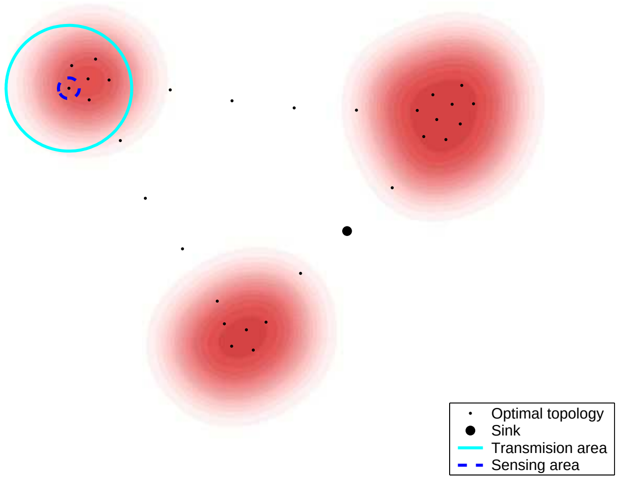

Fig. 7. Convex/large regions scenario, $K = 1$, $\gamma = 0.5$

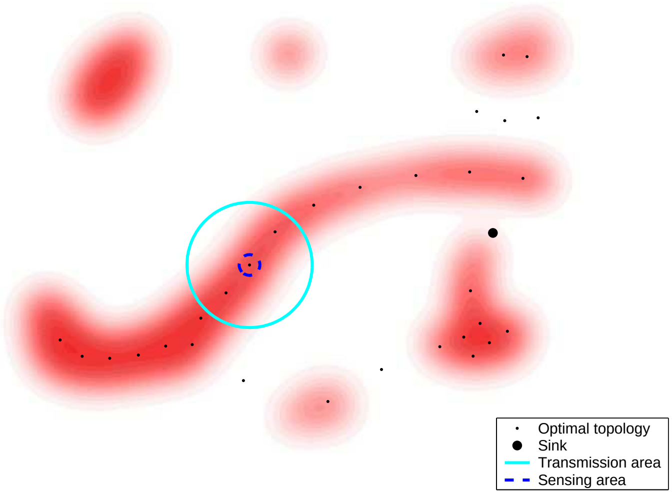

Fig. 8. Non-convex/small regions scenario, $K = 1$, $\gamma = 1.1$

more important, the network can be split due to communication disruptions. Therefore, the information from some node branches may not reach the sinks. Certainly, these effects grow with the variance of the positions. Thus, we are interested in characterizing information loss as a function of that variance.

Specifically, the initially selected position $x = (x^1, x^2)$ of a node is altered by a random variable $(\boldsymbol{X^1}, \boldsymbol{X^2})$. Thus, the final position of the node is $\hat{x} = (x^1 + \boldsymbol{X^1}, x^2 + \boldsymbol{X^2})$. Additionally, let us assume that the modifications of position of different nodes are independent.

Characterizing the random variable $(\boldsymbol{X^1}, \boldsymbol{X^2})$ is a complex task, since it must model many factors (for instance, density of atmospheric layers, wind, height of the launching position, etc.). For simplicity, in this section we will assume that $(\boldsymbol{X^1}, \boldsymbol{X^2})$ is a normal bivariate distribution of null mean, null correlation coefficient (that is, $\boldsymbol{X^1}$ and $\boldsymbol{X^2}$ are independent) and known variance (σ^2). Therefore, σ controls the dispersion of the actual placement from the solution of the global optimization problem.

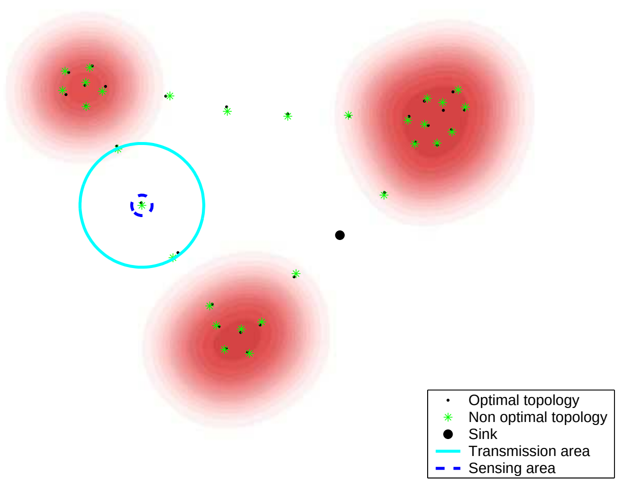

Fig. 9. Convex/large regions scenario, $K = 1$, aerial deployment

Figures 9 and 10 show the effect of aerial deployments on the scenarios of the previous section. Optimal placements are marked as dots, while actual ones are marked as asterisks. In this first example the network is connected, and the variation in information loss is minimal and is exclusively due to the misalignment from the optimal placements. Indeed, figure 11 shows an unconnected network, where the isolated nodes are highlighted. A considerable amount of information is lost (from the isolated branches).

We have also evaluated the effect of σ in the captured importance (α). The following experiment was performed numerically: Starting with the optimal

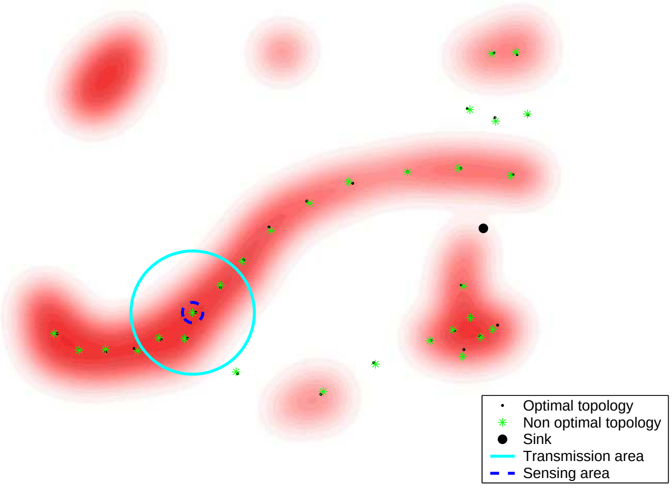

Fig. 10. Non-convex/small regions scenario, $K = 1$, aerial deployment

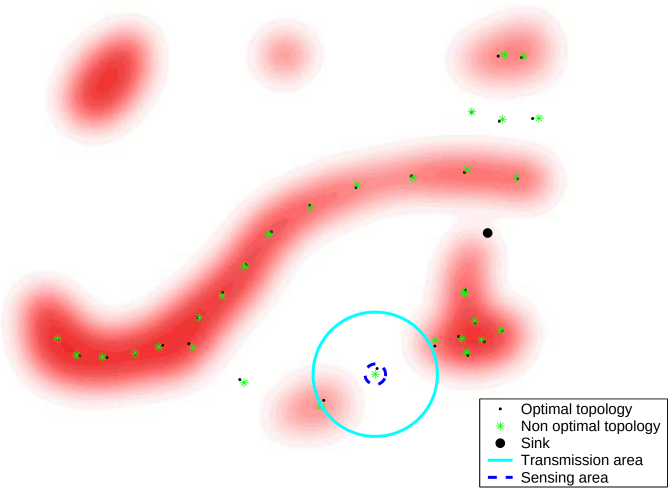

Fig. 11. Convex/large regions scenario, $K = 0$, aerial deployment, unconnected network

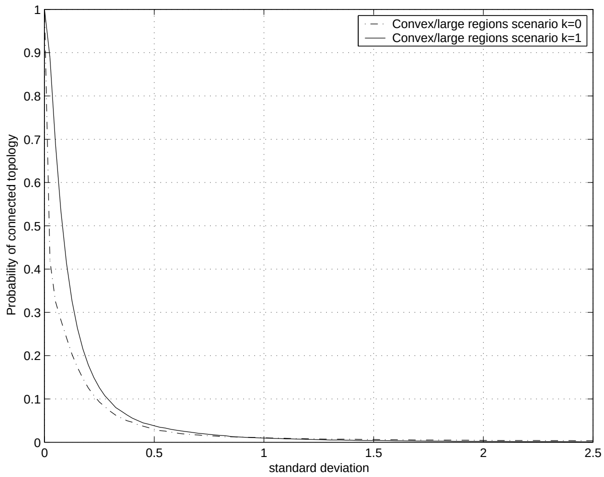

Fig. 12. Convex/large regions scenario, probability of connected topology versus σ

configuration, the position of each node is displaced along both horizontal and vertical axis by two independent normal variables $N(0, \sigma)$. This computation is conducted to obtain statistics of the mean $\hat{\alpha}(X)$ value (see section 3), and the probability that the deployment yields an unconnected network. If the new network is connected (that is, there is a path from every node to the sink) the importance function $\hat{\alpha}(X)$ is computed, and that sample is added to the mean calculation. Otherwise, it is assumed that $\hat{\alpha}(X) = 0$, and the probability of obtaining an unconnected network is updated. This process was performed until the statistics had a quality of 0.95 or better with a relative tolerance of 0.05.

Figures 12 and 13 show the probability that the network is connected for the convex and non-convex scenarios, respectively. As it can be observed, this probability suddenly drops even for low values of σ. Therefore, aerial deployments may produce split networks, and it may be useless to take the optimal configuration as a reference. This effect is consequence of the "bridges" between "information islands", as shown in figure 11. The objective function tends to separate nodes to the maximal distance in zones without importance. This is logical, since a small number of nodes are available, and they must be used to cover the area of interest. However, in the case of aerial deployments, there is a high probability that the network will become unconnected at those bridges.

Besides, figures 14 and 15 show $\hat{\alpha}(X)/\hat{\alpha}(X^*)$ as a function of σ, being X^* the optimal set of locations and X the suboptimal set. That is, the relative importance captured compared against the optimal one. In fact, note that selecting $\sigma = 0$ is equivalent to the optimal configuration in section 5, *i.e.* $X = X^*$. These curves measure both the effect of inaccuracies in the position

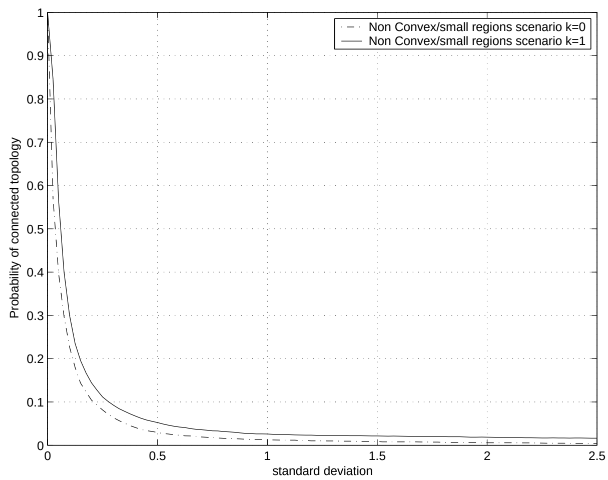

Fig. 13. Non-convex/small regions scenario, probability of connected topology versus σ

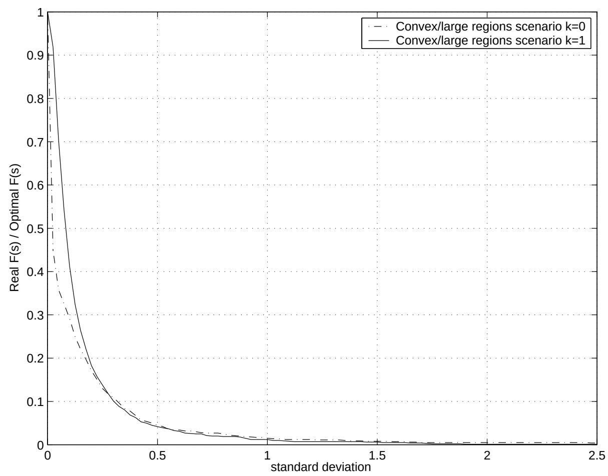

Fig. 14. Convex/large regions scenario, $\hat{\alpha}(X)/\hat{\alpha}(X^*)$ in connected topologies versus σ

of the nodes and the impact of spreading unconnected topologies. As expected, the latter effect rules, producing a fast decreasing of the captured information as σ increases.

We can conclude that the objective function must be redesigned if the scenario contains several separated interest areas to cope with the aerial deployment issue, while still maximizing captured importance.

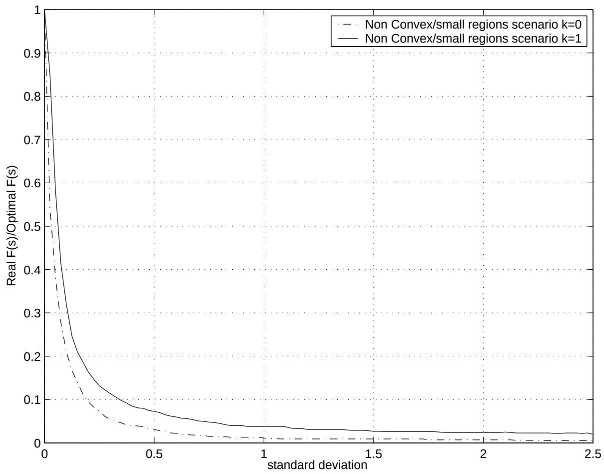

Fig. 15. Non-convex/small regions scenario, $\hat{\alpha}(X)/\hat{\alpha}(X^*)$ in connected topologies versus σ

7 Conclusions and future work

In this work we have developed mathematical programming tools that allow us to select the best placement for a set of nodes in an area of interest. In the formulation of the optimization problem we have modeled the main characteristics of WSNs and some realistic assumptions: the interest is non-uniformly spread over the monitored area, and the network must be connected, i.e. must provide communication paths from every node to at least one of the sinks. The optimal placement is then obtained by means of a simulated annealing algorithm. In addition, we have studied the effect of random variations in the position of the nodes (illustrating an aerial deployment). We have discovered that, albeit it finds optimal placements, our formulation for the optimization problem tends to establish large hop bridges between highly interesting areas. Therefore, small variations in nodes positions may lead to network disconnection, preventing information from isolated nodes to reach the sinks.

Our future work will be oriented towards formulating new optimization procedures that will consider more realistically the inaccuracy of real placements. Indeed, we intend to develop specific solutions for the random placement problem described in the introduction.

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