A Stochastic Approach for Vehicle Safety Modeling in a Platoon of Vehicles Equipped with Vehicular Communications

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***Abstract*—Among the safety-related applications of vehicular ad hoc networks (VANETs), the Cooperative/Chain Collision Avoidance (CCA) techniques have received special attention in recent years. With CCA systems a fast dissemination of warning messages to the vehicles in the platoon enables them to promptly react in emergency situations. In this way the number of car accidents and the associated damage to drivers and passengers can be significantly reduced. These systems are usually evaluated through simulation. However, to cope with the drawbacks of simulating accidents with current tools based on car-following characterizations, we propose the use of a stochastic model as an alternative to simulation for the design and evaluation of such applications. The model allows the computation of the average number of collisions that occur in a platoon of vehicles, the probabilities of the different ways in which the collisions may take place, as well as other statistics of interest. As shown in this paper, the model enables the evaluation of different scenarios and communication technologies, characterized by using the appropriate distributions for its parameters. The main practical utility of this approach lays on its ability to quickly assess numerically the influence of different parameters on the collision process without the need to resort to complex simulations at a first stage. Such an evaluation provides relevant guidelines for the design of vehicular communication systems. To exemplify it, we provide an evaluation of different types of CCA applications in two scenarios, a freeway and an urban scenario.**

***Index Terms*—Vehicle safety, vehicular communications, chain collision, vehicle platoon, collision avoidance, stochastic model, road accidents**

I. INTRODUCTION

Inter-vehicle communications based on wireless technologies can be used for a variety of applications, ranging from comfort and infotainment to traffic safety and driver assistance applications. Improvement of traffic safety by cooperative collision-avoidance (CCA) applications is one of the most promising technical and social benefits of VANETs. With CCA systems, the number of car accidents and their associated damage can be significantly alleviated. However, to design and implement such applications, a deep understanding of the vehicle collision process is needed. The influence of different driving parameters on the collision event must be assessed at an early design stage to develop applications that can timely adapt vehicle dynamics to avoid or at least mitigate the danger.

The safety of a road is signified by the frequency and severity of vehicle crashes that occur in the system in a given period of operation. Safety is related to driver, vehicle and highway operating characteristics. Therefore, it is usually analyzed by studying the influence of these variable operating characteristics on the probability and severity of collisions. In this paper we present a model that can be used to analyze the road safety in many different situations, due to its stochastic nature.

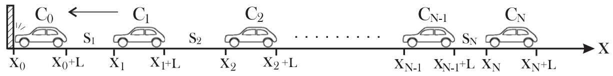

Fig. 1. Initial position of the vehicles in the chain.

In the following sections we present the stochastic model used to evaluate the vehicle safety of a platoon of vehicles (Section II) and compare it with other models found in the open literature (Section III). In Section IV we summarize the main results obtained by the evaluation of different types of CCA applications in a freeway and an urban scenarios. Finally, concluding remarks are presented in Section V.

II. STOCHASTIC MODEL

We consider a road with a single lane and a platoon of N vehicles following a leading one, as in Fig. 1. In this scenario, the proposed model considers the following variables and assumptions:

- The initial state is given by constant velocities $\{V_i\}_{i=0,\ldots,N}$ and inter-vehicle spacings $\{s_i\}_{i=1,\ldots,N}$.
- The leading vehicle faces an emergency situation and immediately brakes at a high deceleration rate, a_0, and sends a warning message to the following vehicles.
- The rest of vehicles brakes at constant deceleration $\{a_i\}_{i=1,\ldots,N}$ after a time lapse $\{\delta_i\}_{i=1,\ldots,N}$.
- All vehicles are equipped with a CCA system.
- Vehicles have the same average length L.
- Vehicles cannot change lane or perform evasive maneuvers (worst case).
- The initial inter-vehicle spacing follows a known probability distribution.

Under these assumptions, the final outcome of a vehicle depends on the outcome of the preceding vehicles. Therefore, the collision model is based on the construction of the probability tree diagram depicted in Fig. 2. We consider an initial state in

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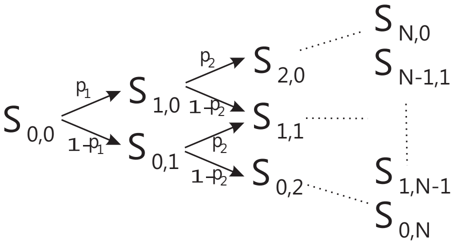


Fig. 2. Probability tree that defines the model. $S_{i,j}$ represents the state with i collided vehicles and j successfully stopped vehicles.

which no vehicle has collided, and in each level of the tree a new vehicle in the chain is considered. From each tree node or state, two possible cases spring, that is, either the following vehicle in the chain may collide (with probability p_i) or stop successfully (with probability $1 - p_i$). At the last level of the probability tree, there are $N + 1$ final outcomes that represent the total number of collided vehicles.

The transition probability between the nodes of the tree is the collision probability of the corresponding vehicle in the chain p_i (or its complementary). These probabilities are crucial to the model and they are computed recursively, as in [1].

A vehicle will collide with its preceding vehicle if the distance it needs to completely stop is greater than the total distance traveled by the preceding vehicle plus the initial inter-vehicle distance. From this observation, and using the average distance traveled by the preceding vehicle, $\overline{l_{i-1}}$, the collision probability of each vehicle C_i (for $i = 1, \ldots, N$) is:

$$p_i = P(d_{s,i} \geq \overline{l_{i-1}} + s_i), \tag{1}$$

where $d_{s,i}$ is the distance needed by vehicle C_i to completely stop, and it is given by:

$$d_{s,i} = \frac{V_i^2}{2a_i} + V_i \delta_i. \tag{2}$$

The $\overline{l_{i-1}}$ computation is not trivial, and it is detailed in [2]. Let us mention that this computation takes into account the different ways a collision may occur (vehicles have not started to brake, only one of them is braking, both of them are braking or the front vehicle has stopped), since each one of these possibilities results in a different distance traveled.

Once the transition probabilities between the nodes of the tree have been obtained, we compute the probabilities of going from the initial state (the time instant when the emergency event occurs) to each final outcome by using Markov chain theory. Finally, if we denote by π_i the probability of reaching the final outcome with i collided vehicles, we compute the average number of collisions in the chain by using the weighted sum:

$$N_{coll} = \sum_{i=0}^{N} i \cdot \pi_i. \tag{3}$$

III. COMPARISON WITH OTHER MODELS

Platoon safety comes as a result of proper stability of the platoon in the presence of perturbations, called string stability. The basis of string stability and safety performance guarantees can be found in [3]. In the absence of safety guarantees collisions may occur and they have been studied mainly by modeling its frequency [4], severity [5] or physical process [6], [7]. In the latter case, very detailed models of vehicle motion and collision dynamics can be found [7], but the equations are completely deterministic, whereas randomness is always present as a real effect of human behavior or noisy operation introduced by on-board sensors or other reasons. To account for it, the usual methodology is to evaluate deterministic models by applying a Monte Carlo or stochastic analysis over an extensive range of their parameters [5], [7]. We are interested in stochastic approaches, so in Table I we briefly describe the models proposed by Choi and Swaroop [8] and by Carbaugh *et al.* [9], and compare them with our model.

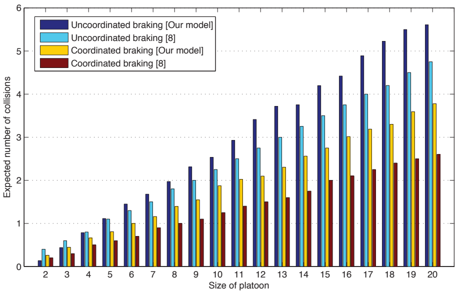


Fig. 3. Average number of collisions for our model and the model in [8].

TABLE II
AVERAGE PERCENTAGE OF ACCIDENTS COMPUTED WITH OUR MODEL AND THE MODEL IN [9].

	Our model	Model in [9]
Manual Unalerted	92.75%	87%
Manual Alerted	14.85%	11%
Automated	6.75%	2.8%

Though there are substantial differences, we have tried to faithfully reproduce their experiments, and the results obtained show a similar trend, as can be seen in Fig. 3 and Table II for the expected number of collisions.

IV. APPLICATIONS OF THE MODEL

In this section we summarize the main results obtained by the thorough evaluation and discussion provided in [10], in which different types of CCA applications in two scenarios (a freeway and an urban scenario) were evaluated. In this evaluation, it was shown that the model can be used at an early stage to shed relevant guidelines for the design of CCA applications, by disclosing the influence of kinematic parameters on the collision process. The details of the characterization of the two scenarios can be found in [10], but they are not provided here because of the lack of space.

First, Fig. 4 shows the influence of isolated controlled parameters on the average number of accidents, by fixing the delay δ, the deceleration rate a or the velocity V separately. The results suggest that the main goal of a CCA application should be to remove the variability of the drivers' reaction time and make cars start braking simultaneously. Hence, although a warning message may help reduce the number of accidents compared to no CCA application at all, to be really effective it needs additional measures involving taking over driver control.

TABLE I

COMPARISON BETWEEN OUR MODEL AND THE MODELS IN [8] AND [9].

	Choi and Swaroop [8]	**Carbaugh et al. [9]**
Description	• Authors assess the benefits of coordination on the safety of a platoon during emergency braking. • They consider that the maximum decelerations of vehicles form a set of independent, identically distributed random variables that follow a known, discrete probability distribution. • The initial following distance and velocity of all vehicles is identical. • The lead vehicle brakes at its maximum capability and the subsequent vehicles obey a vehicle following control law, which can result in a coordinated or uncoordinated braking.	• They quantify the impact of inter-vehicle cooperation and operating speed of an Automated Highway System. • They consider the first rear-end crash (primary collision) experienced by two vehicles that are in the same lane for a significant amount of time prior to the crash. • It is assumed that the lead vehicle brakes at its maximum capability at time 0, and the follower vehicle begins braking at its maximum at time τ. • The modeling parameters can be represented by random variables. • They do not consider the effects of lane changing to avoid collisions.
Metrics	• Severity of impact in terms of relative velocity at collision. • Probability of a primary collision (a collision that occurs irrespective of whether preceding vehicles are involved in a collision). • Expected number of primary collisions.	• Collision velocity distribution (relative velocity of the follower with respect to the leader at the moment of impact). • Primary collisions frequency (total probability associated with the event that the collision velocity is greater than zero). • Primary collisions severity (expected value of the square of the collision velocity conditioned on the occurrence of a collision).
Method	• They assume that the expected relative velocity at impact is the same as the expected relative velocity during primary collisions. • The expected number of total collisions in the string is assumed to be proportional to the expected number of primary collisions. • They assume that a collision occurs whenever the effective deceleration of a vehicle is less than that of the preceding one. • They conduct Monte Carlo simulation over the probability distribution of maximum braking.	• The vehicles are modeled by second order systems with a pure time delay, in the same lane, with no lateral motion. • The collision velocity is calculated kinematically by determining the way in which the collision occurs. • The vehicle parameter distributions are first discretized. For every combination of these discrete parameter values, a collision velocity is calculated using the equations of motion. The probability of that collision velocity is incremented by the probability of the corresponding input parameter value combination.
Differences with our model	• In our model vehicles can travel with different velocities and inter-vehicle spacings. • Our model considers all the types of collisions that can occur in a chain of vehicles, and it gives the probability of each type of collision. • The authors in [8] compute a lower bound of the expected number of accidents, while we compute an upper bound.	• Our model considers all the types of collisions, not only the primary ones. • In our model, the inter-vehicle spacing is introduced as a random variable, not evaluated with Monte Carlo simulations as in this model. • They consider that the inter-vehicle spacing and the relative velocity between two consecutive vehicles are correlated, which is a realistic assumption that we do not consider.

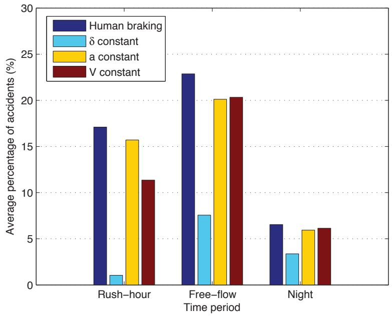
In fact, the warning message delay is not actually relevant to the outcome since it is even in the worst cases, one order of magnitude lower than the driver reaction time. Additionally, for the two cases in the urban scenario and the rush-hour in the freeway, which correspond with low and medium speeds, it seems to be more necessary to control the speed than the deceleration rate.

Secondly, a “more realistic” evaluation, where the system would be able to control several parameters simultaneously, was performed. We compare three CCA systems: brake assist (if the driver has not reacted and started to brake after a given time threshold, the system automatically starts braking at a constant maximum deceleration); automatic braking (when the vehicle receives the warning message it immediately starts to brake, although the driver is not yet aware of the risk); and automatic braking + speed control (a proactive CCA mechanism that controls the speed of vehicles before the emergency situation occurs, besides allowing the automatic braking of the vehicles in the chain). In addition to the human braking, we take into account a more realistic situation in which the driver adapts its deceleration rate to the velocity and deceleration of the preceding vehicle (human braking + deceleration adaptation).

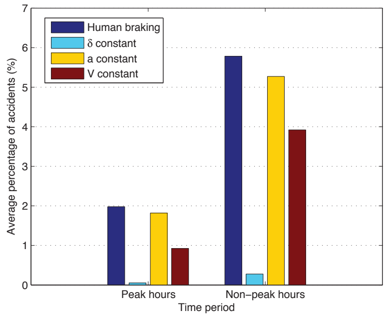
Looking at Fig. 5, it can be noticed that the percentage of accidents in the case of rational deceleration (human braking + deceleration adaptation) is much lower, more than 50%, than when it is not considered. But even considering this human braking behavior, more sophisticated CCA applications still reduce the percentage of accidents.

Brake assist characterizes approximately current systems and is the most likely CCA to be deployed in the near future. Its performance is between rational human and automatic (coordinated) braking as could also be expected, but still much closer to human braking, showing that there are still potential gains from enabling systems with more cooperation, as automatic braking. In fact, these gains are dependent on the reaction threshold, which should be carefully selected.

It should be noted that particular numerical results have to be considered as upper bounds on the expected number of accidents, since our model is based on the strong assumption that vehicles cannot change lane to avoid the crash (worst case). However, even for the generic scenarios and simplified systems used as examples, it is able to provide a reasonable qualitative insight of the relative benefits of different CCA approaches.

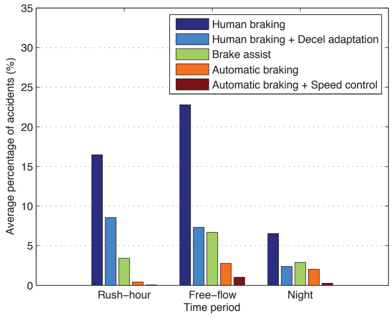


(a) Freeway Scenario.

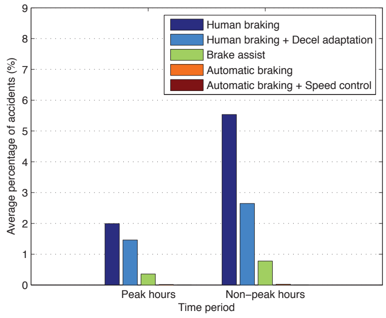


(b) Urban Scenario.

Fig. 4. Influence of the principal kinematic parameters on the average percentage of accidents for the scenarios under evaluation.



(a) Freeway Scenario.



(b) Urban Scenario.

Fig. 5. Average percentage of accidents for different CCA proposals in the scenarios under evaluation.

V. CONCLUSIONS

In this paper we have presented a safety modeling methodology via a stochastic model, and we have compared it with other safety models, focusing on two stochastic approaches. In opposition to these two models, our model takes into account not only the primary collisions in the platoon, but also all the secondary collisions triggered by them. Moreover, we compute the probability of occurrence of each type of collision, in addition to the usual safety measures, namely, the expected number and severity of collisions. As an important novelty, it should be noted that the inter-vehicle spacing is introduced in our model as a random variable, not evaluated via Monte Carlo simulations, as it is usually done.

Finally, we have exemplified the utility of our model to numerically evaluate different types of CCA applications. The results suggest that the variability due to the drivers reaction time is the primary cause of accidents and so removing it should be the main focus of a CCA application.

However, we must say that our model has some limitations. In its current form, it cannot capture state variable correlations, which decrease the model accuracy. Therefore, the numerical results should be considered as overestimations of the actual values. Nevertheless, we intend to undertake this challenge in the near future.

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